ECON 452*: *Stata 11* Tutorial 9

TOPIC: **Estimating and Interpreting Probit Models with *Stata*: Extensions**

DATA: **mroz.dta** (a *Stata*-format dataset you created in ***Stata 11 Tutorial 8***)

TASKS: ***Stata 11 Tutorial 9*** is an extension of *Stata 11 Tutorial 8*, and therefore deals with the estimation, testing, and interpretation of ***probit models*** for binary dependent variables. In particular, it illustrates how to use a cross-sectional sample of married women in the United States to investigate ***whether* and *how* the probability of labour force participation differs between two distinct groups of married women, namely (1) married women who have one or more pre-school aged children and (2) married women who have no pre-school aged children.** It demonstrates how *Stata* can be used to conduct an econometric investigation into differences in the conditional probability of labour force participation between these two distinct groups of married women.

- The ***Stata* commands** that constitute the primary subject of this tutorial are:

probit Used to compute ML estimates of ***probit* coefficients** in probit models of binary dependent variables.

dprobit Used to compute ML estimates of the **marginal *probability* effects** of explanatory variables in probit models.

test Used after probit estimation to compute ***Wald tests*** of linear coefficient equality restrictions on probit coefficients.

lincom Used after probit estimation to compute and test the marginal effects of individual explanatory variables.

- The ***Stata*** **statistical functions** used in this tutorial are:

normalden(*z*)	Computes ***value of the standard normal <u>density</u> function (p.d.f.)*** for a given value ***z*** of a standard normal random variable.
normal(*z*)	Computes ***value of the standard normal <u>distribution</u> function (c.d.f.)*** for a given value ***z*** of a standard normal random variable.
invnormal(*p*)	Computes ***the inverse of the standard normal <u>distribution</u> function;*** if **normal(*z*) = *p*, then invnormal(*p*) = *z***.

NOTE: *Stata* commands are *case sensitive*. All *Stata command names* must be typed in the Command window in ***lower case* letters**.

<u>Lecture Notes on *Stata 11* Tutorial 9</u>: CONTENTS

❑ Two Probit Models of Married Women's Participation: Specification of Models 2 and 3

We consider two different models of married women's labour force participation.

- **Model 2** was introduced in *Stata 11 Tutorial 8*. The **binary indicator variable *$dkidslt6_i$*** enters only as an **additive regressor**.
- **Model 3** is a generalization of Model 2: it allows all probit coefficients to differ between (1) married women who currently have one or more pre-school aged children and (2) married women who currently have no pre-school aged children. The **binary explanatory variable *$dkidslt6_i$*** enters both **additively and multiplicatively**.

The ***observed dependent variable*** in both models is the binary variable ***$inlf_i$*** defined as follows:

$inlf_i$ = 1 if the i-th married woman is in the employed labour force
= 0 if the i-th married woman is not in the employed labour force

The ***explanatory variables*** in Models 2 and 3 are:

$nwifeinc_i$ = non-wife family income of the i-th woman (in thousands of dollars per year);

ed_i = years of formal education of the i-th woman (in years);

exp_i = years of actual work experience of the i-th woman (in years);

age_i = age of the i-th woman (in years);

$dkidslt6_i$ = 1 if the i-th woman has one or more children less than 6 years of age, = 0 otherwise.

Four of these explanatory variables -- $nwifeinc_i$, ed_i, exp_i, and age_i -- are ***continuous*** variables, whereas the fifth explanatory variable -- $dkidslt6_i$ -- is a ***binary*** indicator (dummy) variable.

<u>Model 2</u> – binary explanatory variable *dkidslt6$_i$* enters only additively

The **probit index function for Model 2** is:

$$x_i^T\beta = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i + \beta_6 dkidslt6_i$$

Remarks: In Model 2, the binary explanatory variable $dkidslt6_i$ enters only additively; only the intercept coefficient in the index function differs between the two groups of married women, those who have pre-school aged children and those who do not.

- In Model 2, the probit index function for ***married women who have no pre-school aged children***, for whom $dkidslt6_i = 0$, is obtained by setting $dkidslt6_i = 0$ in the index function for Model 2:

$$\left(x_i^T\beta \,\middle|\, dkidslt6_i = 0\right) = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i + \beta_6 0$$

$$= \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i$$

- In Model 2, the probit index function for ***married women who have one or more pre-school aged children***, for whom $dkidslt6_i = 1$, is obtained by setting $dkidslt6_i = 1$ in the index function for Model 2:

$$\left(x_i^T\beta \,\middle|\, dkidslt6_i = 1\right) = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i + \beta_6 1$$

$$= \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i + \beta_6$$

- In Model 2, the **marginal *index* effect** of the **binary indicator variable *dkidslt6i*** is simply the difference between (1) the index function for ***married women who currently have one or more pre-school aged children***, $\left(x_i^T\beta \mid \text{dkidslt6}_i = 1\right)$ and (2) the index function for ***married women who currently have no pre-school aged children***, $\left(x_i^T\beta \mid \text{dkidslt6}_i = 0\right)$:

$$\left(x_i^T\beta \mid \text{dkidslt6}_i = 1\right) - \left(x_i^T\beta \mid \text{dkidslt6}_i = 0\right)$$

$$= \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6$$
$$- \left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)$$

$$= \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6$$
$$- \beta_0 - \beta_1\text{nwifeinc}_i - \beta_2\text{ed}_i - \beta_3\exp_i - \beta_4\exp_i^2 - \beta_5\text{age}_i$$

$$= \beta_6$$

- In Model 2, the **marginal *probability* effect** of the **binary indicator variable *dkidslt6$_i$*** is the difference between (1) the conditional probability that **inlf$_i$** = 1 for ***married women with one or more pre-school aged children*** and (2) the conditional probability that **inlf$_i$** = 1 for ***married women with no pre-school aged children***:

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) - \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) = \Phi\left(x_{1i}^T\beta\right) - \Phi\left(x_{0i}^T\beta\right)$$

where $\Phi(*)$ is the cumulative distribution function (cdf) of the standard normal distribution and

$$x_{1i}^T = \left(1 \;\; \text{nwifeinc}_i \;\; \text{ed}_i \;\; \exp_i \;\; \exp_i^2 \;\; \text{age}_i \;\; 1\right)$$

$$x_{0i}^T = \left(1 \;\; \text{nwifeinc}_i \;\; \text{ed}_i \;\; \exp_i \;\; \exp_i^2 \;\; \text{age}_i \;\; 0\right)$$

$$\beta = \left(\beta_0 \;\; \beta_1 \;\; \beta_2 \;\; \beta_3 \;\; \beta_4 \;\; \beta_5 \;\; \beta_6\right)^T$$

$$x_{1i}^T\beta = \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6$$

$$x_{0i}^T\beta = \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i$$

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) = \Phi\left(x_{1i}^T\beta\right) = \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6\right)$$

$$\begin{aligned}\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) = \Phi\left(x_{0i}^T\beta\right) &= \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6 0\right) \\ &= \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)\end{aligned}$$

Thus, the **marginal *probability* effect of the indicator variable *dkidslt6$_i$* in Model 2** is

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) - \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) = \Phi\left(x_{1i}^T\beta\right) - \Phi\left(x_{0i}^T\beta\right)$$

$$= \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6\right)$$

$$- \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)$$

<u>Model 3</u> – a full interaction model in the binary variable *dkidslt6*

The **probit index function**, or regression function, **for Model 3** is:

$$x_i^T\beta = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i + \beta_6 dkidslt6_i$$
$$+ \beta_7 dkidslt6_i\, nwifeinc_i + \beta_8 dkidslt6_i\, ed_i + \beta_9 dkidslt6_i \exp_i + \beta_{10} dkidslt6_i \exp_i^2 + \beta_{11} dkidslt6_i\, age_i$$

Remarks: Model 3 is the ***full-interaction*** generalization of Model 2: it interacts the $dkidslt6_i$ indicator variable with all the other regressors in Model 2, and thereby permits all index function coefficients to differ between the two groups of married women distinguished by $dkidslt6_i$.

- In Model 3, the **probit *index* function** for ***married women who currently have no pre-school aged children***, for whom $dkidslt6_i = 0$, is obtained by setting $dkidslt6_i = 0$ in the index function for Model 3:

$$\left(x_i^T\beta \,\middle|\, dkidslt6_i = 0\right) = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i$$

- In Model 3, the **probit *index* function** for ***married women who currently have one or more pre-school aged children***, for whom $dkidslt6_i = 1$, is obtained by setting $dkidslt6_i = 1$ in the index function for Model 3:

$$\left(x_i^T\beta \,\middle|\, dkidslt6_i = 1\right) = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i$$
$$+ \beta_6 1 + \beta_7 1\cdot nwifeinc_i + \beta_8 1\cdot ed_i + \beta_9 1\cdot \exp_i + \beta_{10} 1\cdot \exp_i^2 + \beta_{11} 1\cdot age_i$$
$$= \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i + \beta_6 + \beta_7 nwifeinc_i + \beta_8 ed_i + \beta_9 \exp_i + \beta_{10} \exp_i^2 + \beta_{11} age_i$$
$$= \beta_0 + \beta_6 + (\beta_1 + \beta_7) nwifeinc_i + (\beta_2 + \beta_8) ed_i + (\beta_3 + \beta_9) \exp_i + (\beta_4 + \beta_{10}) \exp_i^2 + (\beta_5 + \beta_{11}) age_i$$

- In Model 3, the **marginal *index* effect** of the **binary indicator variable *dkidslt6ᵢ*** is simply the difference between (1) the index function for ***married women who currently have one or more pre-school aged children***, $\left(x_i^T\beta \middle| \text{dkidslt6}_i = 1\right)$ and (2) the index function for ***married women who currently have no pre-school aged children***, $\left(x_i^T\beta \middle| \text{dkidslt6}_i = 0\right)$:

$$\left(x_i^T\beta \middle| \text{dkidslt6}_i = 1\right) - \left(x_i^T\beta \middle| \text{dkidslt6}_i = 0\right)$$

$$= \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i$$
$$- \left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)$$

$$= \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i$$
$$- \beta_0 - \beta_1\text{nwifeinc}_i - \beta_2\text{ed}_i - \beta_3\exp_i - \beta_4\exp_i^2 - \beta_5\text{age}_i$$

$$= \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i$$

- In Model 3, the **marginal *probability* effect** of the **binary indicator variable *dkidslt6ᵢ*** is the difference between (1) the conditional probability that **$inlf_i$** = 1 for ***married women with one or more pre-school aged children*** and (2) the conditional probability that **$inlf_i$** = 1 for ***married women with no pre-school aged children***:

$$\Pr\left(inlf_i = 1 \middle| dkidslt6_i = 1\right) - \Pr\left(inlf_i = 1 \middle| dkidslt6_i = 0\right) = \Phi\left(x_{1i}^T\beta\right) - \Phi\left(x_{0i}^T\beta\right)$$

where $\Phi(*)$ is the cumulative distribution function (cdf) of the standard normal distribution and

$$x_{1i}^T = \left(1\ \ nwifeinc_i\ \ ed_i\ \ \exp_i\ \ \exp_i^2\ \ age_i\ \ 1\ \ nwifeinc_i\ \ ed_i\ \ \exp_i\ \ \exp_i^2\ \ age_i\right)$$

$$x_{0i}^T = \left(1\ \ nwifeinc_i\ \ ed_i\ \ \exp_i\ \ \exp_i^2\ \ age_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\right)$$

$$\beta = \left(\beta_0\ \ \beta_1\ \ \beta_2\ \ \beta_3\ \ \beta_4\ \ \beta_5\ \ \beta_6\ \ \beta_7\ \ \beta_8\ \ \beta_9\ \ \beta_{10}\ \ \beta_{11}\right)^T$$

$$\begin{aligned} x_{1i}^T\beta = {} & \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i \\ & + \beta_6 + \beta_7 nwifeinc_i + \beta_8 ed_i + \beta_9 \exp_i + \beta_{10} \exp_i^2 + \beta_{11} age_i \end{aligned}$$

$$x_{0i}^T\beta = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i$$

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) = \Phi\begin{pmatrix} \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i \\ + \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i \end{pmatrix}$$

$$= \Phi\begin{pmatrix} (\beta_0 + \beta_6) + (\beta_1 + \beta_7)\text{nwifeinc}_i + (\beta_2 + \beta_8)\text{ed}_i \\ + (\beta_3 + \beta_9)\exp_i + (\beta_4 + \beta_{10})\exp_i^2 + (\beta_5 + \beta_{11})\text{age}_i \end{pmatrix}$$

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) = \Phi\begin{pmatrix} \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i \\ + \beta_6 0 + \beta_7 0 + \beta_8 0 + \beta_9 0 + \beta_{10} 0 + \beta_{11} 0 \end{pmatrix}$$

$$= \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)$$

Thus, the **marginal *probability* effect of the indicator variable *dkidslt6$_i$*** in Model 3 is

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) - \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) =$$

$$\Phi\begin{pmatrix} \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i \\ + \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i \end{pmatrix}$$
$$- \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)$$

We are concerned with **three aspects** of the **marginal *probability* effect of the indicator variable *dkidslt6$_i$***:

1. the **<u>existence</u>** of the **marginal *probability* effect of the indicator variable *dkidslt6$_i$***;
2. the **<u>direction</u> (sign)** of the **marginal *probability* effect of the indicator variable *dkidslt6$_i$***;
3. the **<u>magnitude</u> (size)** of the **marginal *probability* effect of the indicator variable *dkidslt6$_i$***.

❑ Testing the marginal *probability* effect of the binary explanatory variable $dkidslt6_i$ -- *test* and *lincom*

Proposition to be Tested

- Does the ***conditional*** **probability of labour force participation** for married women depend on the presence in the family of one or more dependent children under 6 years of age?
- Is the probability of labour force participation for married women with given values of nwifeinc_i, ed_i, exp_i, and age_i who currently have one or more pre-school aged children equal to the probability of labour force participation for married women with the same values of nwifeinc_i, ed_i, exp_i, and age_i who currently have no pre-school aged children?
- Is it true that

$$\begin{aligned}&\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1,\ \text{nwifeinc}_i,\ \text{ed}_i,\ \text{exp}_i,\ \text{age}_i\right)\\ &\quad = \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0,\ \text{nwifeinc}_i,\ \text{ed}_i,\ \text{exp}_i,\ \text{age}_i\right)?\end{aligned}$$

Null and Alternative Hypotheses: General Formulation

The ***null hypothesis*** in general is:

$$H_0:\quad \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1,\ \ldots\right) = \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0,\ \ldots\right)$$

The ***alternative hypothesis*** in general is:

$$H_1:\quad \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1,\ \ldots\right) \neq \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0,\ \ldots\right)$$

Testing the <u>Existence</u> of the Marginal Probability Effect of the Indicator Variable *dkidslt6* $dkidslt6_i$

For testing the ***existence*** **of a relationship** between any explanatory variable and the probability that the observed dependent variable equals 1, use either of the two *Stata* commands for probit estimation: use ***either*** the **probit** command ***or*** the **dprobit** command.

<u>Null and Alternative Hypotheses: Model 2</u>

The null hypothesis in general is:

$$H_0: \quad \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1, \ \ldots\right) = \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0, \ \ldots\right)$$

For Model 2,

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1, \ \ldots\right) = \Phi\left(x_i^T\beta \middle| \text{dkidslt6}_i = 1\right)$$
$$= \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5\text{age}_i + \beta_6\right)$$

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0, \ \ldots\right) = \Phi\left(x_i^T\beta \middle| \text{dkidslt6}_i = 0\right)$$
$$= \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5\text{age}_i\right)$$

These two probabilities are equal if the exclusion restriction $\beta_6 = 0$ is true. In other words, **a *sufficient* condition** for these two probabilities to be equal is the exclusion restriction $\boldsymbol{\beta_6 = 0}$.

The ***null* and *alternative* hypotheses for Model 2** are therefore:

H_0: $\beta_6 = 0$
H_1: $\beta_6 \neq 0$

Important Point: A test of the null hypothesis that the **marginal *probability* effect** of pre-school aged children is zero **is equivalent to** a test of the null hypothesis that the **marginal *index* effect** of pre-school aged children is zero.

- **Marginal *probability* effect of <u>pre-school aged children</u> equals *zero*** in Model 2 if

$$\Phi\left(x_i^T\beta \mid \text{dkidslt6}_i = 1\right) = \Phi\left(x_i^T\beta \mid \text{dkidslt6}_i = 0\right).$$

In Model 2,

$$\Phi\left(x_i^T\beta \mid \text{dkidslt6}_i = 1\right) = \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i + \beta_6\right)$$

$$\Phi\left(x_i^T\beta \mid \text{dkidslt6}_i = 0\right) = \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)$$

Question: What coefficient restriction(s) are sufficient to make these two probabilities equal for any given values of nwifeinc_i, ed_i, $\exp_i$, and age_i?

Answer: By inspection – i.e., by comparing the function $\Phi\left(x_i^T\beta \mid \text{dkidslt6}_i = 1\right)$ and the function $\Phi\left(x_i^T\beta \mid \text{dkidslt6}_i = 0\right)$ – we can see that a sufficient condition for $\Phi\left(x_i^T\beta \mid \text{dkidslt6}_i = 1\right) = \Phi\left(x_i^T\beta \mid \text{dkidslt6}_i = 0\right)$ in Model 2 is the single coefficient exclusion restriction $\beta_6 = 0$.

- **Marginal *index* effect of <u>pre-school aged children</u> equals *zero*** if

  $\left(x_i^T\beta \mid dkidslt6_i = 1\right) = \left(x_i^T\beta \mid dkidslt6_i = 0\right)$.

  In Model 2,

  $$\left(x_i^T\beta \mid dkidslt6_i = 1\right) = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i + \beta_6$$

  $$\left(x_i^T\beta \mid dkidslt6_i = 0\right) = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i$$

  Question: What coefficient restriction(s) are sufficient to make these two index functions equal for any given values of $nwifeinc_i$, ed_i, $\exp_i$, and age_i?

  Answer: By inspection – i.e., by comparing the index function $\left(x_i^T\beta \mid dkidslt6_i = 1\right)$ and the index function $\left(x_i^T\beta \mid dkidslt6_i = 0\right)$ – we can see that a sufficient condition for $\left(x_i^T\beta \mid dkidslt6_i = 1\right) = \left(x_i^T\beta \mid dkidslt6_i = 0\right)$ in Model 2 is the single coefficient exclusion restriction $\beta_6 = 0$.

- ***<u>Result</u>:*** The single coefficient exclusion restriction $\beta_6 = 0$ is sufficient to make the ***both* the marginal *probability* effect *and* the marginal *index* effect** of pre-school aged children equal to zero in Model 2.

❑ How to Perform this Test for Model 2 in *Stata*

- First, compute ML estimates of probit Model 2 and display the full set of saved results. Enter the following commands:

```
probit inlf nwifeinc ed exp expsq age dkidslt6
ereturn list
```

- To calculate a **Wald test** of H_0 against H_1 and the p-value for the calculated W-statistic, enter the following **test**, **return list** and **display** commands:

```
test dkidslt6     or   test dkidslt6 = 0
return list
display sqrt(r(chi2))
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following **lincom**, **return list** and **display** commands:

```
lincom _b[dkidslt6]
return list
display r(estimate)/r(se)
```

The results of this two-tail t-test are identical with those of the previous Wald test.

Note that this **lincom** command merely replicates the test statistic and p-value that are displayed in the output of the **probit** command for the regressor ***dkidslt6***.

Null and Alternative Hypotheses: Model 3

The null hypothesis in general is:

$$H_0: \quad \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1, \ldots\right) = \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0, \ldots\right)$$

For Model 3,

$$\begin{aligned}\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) &= \Phi\left(x_i^T\beta \middle| \text{dkidslt6}_i = 1\right) \\ &= \Phi\begin{pmatrix}\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i \\ + \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) &= \Phi\left(x_i^T\beta \middle| \text{dkidslt6}_i = 0\right) \\ &= \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)\end{aligned}$$

These two probabilities are equal if the six exclusion restrictions $\beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = 0$ are true. In other words, a ***sufficient condition*** for **these two probabilities to be *equal*** is the set of six coefficient exclusion restrictions $\beta_j = 0$ for all j = 6, …, 11.

The ***null* and *alternative* hypotheses for Model 3** are therefore:

H_0: $\beta_j = 0 \quad \forall \; j = 6, 7, 8, 9, 10, 11$

$\Rightarrow \quad \beta_6 = 0$ *and* $\beta_7 = 0$ *and* $\beta_8 = 0$ *and* $\beta_9 = 0$ *and* $\beta_{10} = 0$ *and* $\beta_{11} = 0$

H_1: $\beta_j \neq 0 \qquad j = 6, 7, 8, 9, 10, 11$

$\Rightarrow \quad \beta_6 \neq 0$ *and/or* $\beta_7 \neq 0$ *and/or* $\beta_8 \neq 0$ *and/or* $\beta_9 \neq 0$ *and/or* $\beta_{10} \neq 0$ *and/or* $\beta_{11} \neq 0$

Important Point: A test of the null hypothesis that the **marginal *probability* effect** of pre-school aged children is zero **is equivalent to** a test of the null hypothesis that the **marginal *index* effect** of pre-school aged children is zero.

- **<u>Marginal *probability* effect</u> of pre-school aged children equals *zero*** in Model 3 if

$\Phi\left(x_i^T\beta \,\middle|\, dkidslt6_i = 1\right) = \Phi\left(x_i^T\beta \,\middle|\, dkidslt6_i = 0\right)$.

In Model 3,

$$\Phi\left(x_i^T\beta \,\middle|\, dkidslt6_i = 1\right) = \Phi\begin{pmatrix}\beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i \\ + \beta_6 + \beta_7 nwifeinc_i + \beta_8 ed_i + \beta_9 \exp_i + \beta_{10} \exp_i^2 + \beta_{11} age_i\end{pmatrix}$$

$$\Phi\left(x_i^T\beta \,\middle|\, dkidslt6_i = 0\right) = \Phi\left(\beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i\right)$$

Question: What coefficient restriction(s) are sufficient to make these two probabilities equal for any given values of $nwifeinc_i$, ed_i, $\exp_i$, and age_i?

Answer: By inspection – i.e., by comparing the function $\Phi\left(x_i^T\beta \,\middle|\, dkidslt6_i = 1\right)$ and the function $\Phi\left(x_i^T\beta \,\middle|\, dkidslt6_i = 0\right)$ – we can see that a sufficient condition for $\Phi\left(x_i^T\beta \,\middle|\, dkidslt6_i = 1\right) =$ $\Phi\left(x_i^T\beta \,\middle|\, dkidslt6_i = 0\right)$ in Model 3 is the set of ***six* coefficient exclusion restrictions** $\boldsymbol{\beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = 0}$.

- **<u>Marginal *index* effect</u> of pre-school aged children equals *zero*** in Model 3 if

  $\left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 1\right) = \left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 0\right)$.

  In Model 3,

  $$\left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 1\right) = \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i$$
  $$+ \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i$$

  $$\left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 0\right) = \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i$$

  Question: What coefficient restriction(s) are sufficient to make these two index functions equal for any given values of nwifeinc_i, ed_i, $\exp_i$, and age_i?

  Answer: By inspection – i.e., by comparing the index function $\left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 1\right)$ and the index function $\left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 0\right)$ – we can see that a sufficient condition for $\left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 1\right) = \left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 0\right)$ in Model 3 is the set of ***six* coefficient exclusion restrictions $\beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = 0$.**

- ***<u>Result</u>:*** The six coefficient exclusion restrictions **$\beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = 0$** are sufficient to make the ***both* the marginal *probability* effect *and* the marginal *index* effect** of pre-school aged children **equal to *zero*** in Model 3.

❑ How to Perform this Test for Model 3 in *Stata*

H_0: $\beta_j = 0 \quad \forall \; j = 6, 7, 8, 9, 10, 11$

H_1: $\beta_j \neq 0 \qquad j = 6, 7, 8, 9, 10, 11$

- Before estimating Model 3, it is necessary to create the ***dkidslt6$_i$* interaction variables**. Enter the following **generate** commands:

```
generate d6nwinc = dkidslt6*nwifeinc
generate d6ed = dkidslt6*ed
generate d6exp = dkidslt6*exp
generate d6expsq = dkidslt6*expsq
generate d6age = dkidslt6*age
```

- Next, compute ML estimates of probit Model 3 and display the full set of saved results. Enter the following commands:

```
probit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age
ereturn list
```

- To calculate a **Wald test** of H_0 against H_1 and the p-value for the calculated W-statistic, enter the following **test** and **return list** commands:

```
test dkidslt6 d6nwinc d6ed d6exp d6expsq d6age
return list
```

- A second hypothesis test you should perform on Model 3 is a test of the null hypothesis that ***all slope* coefficient differences** between married women who have one or more pre-school aged children and married women who have no pre-school aged children **equal zero**. The null and alternative hypotheses are:

  H_0: $\beta_j = 0 \quad \forall \; j = 7, 8, 9, 10, 11$

  $\Rightarrow \quad \beta_7 = 0 \textit{ and } \beta_8 = 0 \textit{ and } \beta_9 = 0 \textit{ and } \beta_{10} = 0 \textit{ and } \beta_{11} = 0$

  H_1: $\beta_j \neq 0 \quad j = 7, 8, 9, 10, 11$

  $\Rightarrow \quad \beta_7 \neq 0 \textit{ and/or } \beta_8 \neq 0 \textit{ and/or } \beta_9 \neq 0 \textit{ and/or } \beta_{10} \neq 0 \textit{ and/or } \beta_{11} \neq 0$

  Note that the null hypothesis H_0 implies Model 2, whereas the alternative hypothesis H_1 implies Model 3. Enter the **test** command:

```
test d6nwinc d6ed d6exp d6expsq d6age
```

```
. probit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age

Iteration 0:   log likelihood =  -514.8732
Iteration 1:   log likelihood = -406.48086
Iteration 2:   log likelihood = -402.63328
Iteration 3:   log likelihood = -402.61111
Iteration 4:   log likelihood = -402.61111

Probit estimates                                  Number of obs   =        753
                                                  LR chi2(11)     =     224.52
                                                  Prob > chi2     =     0.0000
Log likelihood = -402.61111                       Pseudo R2       =     0.2180

------------------------------------------------------------------------------
        inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    nwifeinc |  -.0109103   .0056007    -1.95   0.051    -.0218874    .0000668
          ed |   .1215786   .0280427     4.34   0.000     .0666159    .1765413
         exp |    .137317   .0208939     6.57   0.000     .0963657    .1782682
       expsq |  -.0022349   .0006495    -3.44   0.001     -.003508   -.0009619
         age |  -.0593504   .0085496    -6.94   0.000    -.0761072   -.0425935
    dkidslt6 |  -2.527031   1.267708    -1.99   0.046    -5.011694   -.0423684
     d6nwinc |  -.0059201   .0109624    -0.54   0.589    -.0274059    .0155658
        d6ed |   .0327202   .0623143     0.53   0.600    -.0894135     .154854
       d6exp |  -.1128835   .0663563    -1.70   0.089    -.2429394    .0171724
     d6expsq |   .0030026   .0033465     0.90   0.370    -.0035564    .0095616
       d6age |   .0503914   .0260813     1.93   0.053    -.0007271    .1015099
       _cons |   .6084091   .4961565     1.23   0.220    -.3640398    1.580858
------------------------------------------------------------------------------
```

```
. ereturn list

scalars:
                 e(N) =  753
              e(ll_0) =  -514.8732045671461
                e(ll) =  -402.6111063731551
              e(df_m) =  11
              e(chi2) =  224.5241963879821
              e(r2_p) =  .2180383387563736

macros:
            e(depvar) : "inlf"
               e(cmd) : "probit"
          e(crittype) : "log likelihood"
           e(predict) : "probit_p"
          e(chi2type) : "LR"

matrices:
                 e(b) :  1 x 12
                 e(V) :  12 x 12

functions:
            e(sample)
```

```
. * Test 1:
. test dkidslt6 d6nwinc d6ed d6exp d6expsq d6age

 ( 1)  dkidslt6 = 0
 ( 2)  d6nwinc = 0
 ( 3)  d6ed = 0
 ( 4)  d6exp = 0
 ( 5)  d6expsq = 0
 ( 6)  d6age = 0

           chi2(  6) =   58.11
         Prob > chi2 =    0.0000

. return list

scalars:
              r(drop) =  0
              r(chi2) =  58.11036668348744
                r(df) =  6
                 r(p) =  1.08838734793e-10
```

```
. * Test 2:
. test d6nwinc d6ed d6exp d6expsq d6age

 ( 1)  d6nwinc = 0
 ( 2)  d6ed = 0
 ( 3)  d6exp = 0
 ( 4)  d6expsq = 0
 ( 5)  d6age = 0

           chi2(  5) =    9.03
         Prob > chi2 =    0.1078

. return list

scalars:
              r(drop) =  0
              r(chi2) =  9.031191992371875
                r(df) =  5
                 r(p) =  .1078264635420236
```

```
. dprobit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age

Iteration 0:   log likelihood =  -514.8732
Iteration 1:   log likelihood = -406.48086
Iteration 2:   log likelihood = -402.63328
Iteration 3:   log likelihood = -402.61111
Iteration 4:   log likelihood = -402.61111

Probit estimates                                      Number of obs =    753
                                                      LR chi2(11)   = 224.52
                                                      Prob > chi2   = 0.0000
Log likelihood = -402.61111                           Pseudo R2     = 0.2180

------------------------------------------------------------------------------
    inlf |      dF/dx   Std. Err.      z    P>|z|     x-bar  [    95% C.I.   ]
---------+--------------------------------------------------------------------
nwifeinc |  -.0042484   .0021794    -1.95   0.051    20.129   -.00852  .000023
      ed |   .0473425   .0108958     4.34   0.000   12.2869   .025987  .068698
     exp |    .053471   .0081365     6.57   0.000   10.6308   .037524  .069418
   expsq |  -.0008703   .0002531    -3.44   0.001   178.039  -.001366 -.000374
     age |  -.0231109   .0033213    -6.94   0.000   42.5378  -.029621 -.016601
dkidslt6*|  -.7273305   .1555487    -1.99   0.046   .195219   -1.0322 -.422461
 d6nwinc |  -.0023053     .00427    -0.54   0.589   4.04408  -.010674  .006064
    d6ed |   .0127412   .0242742     0.53   0.600   2.47809  -.034835  .060318
   d6exp |  -.0439567   .0258347    -1.70   0.089   1.37317  -.094592  .006678
 d6expsq |   .0011692   .0013032     0.90   0.370    15.012  -.001385  .003723
   d6age |   .0196223   .0101508     1.93   0.053   6.87251  -.000273  .039518
---------+--------------------------------------------------------------------
  obs. P |   .5683931
 pred. P |   .5870885  (at x-bar)
------------------------------------------------------------------------------
(*) dF/dx is for discrete change of dummy variable from 0 to 1
    z and P>|z| are the test of the underlying coefficient being 0
```

```
. ereturn list

scalars:
                  e(N) =  753
               e(ll_0) =  -514.8732045671461
                 e(ll) =  -402.6111063731551
               e(df_m) =  11
               e(chi2) =  224.5241963879821
               e(r2_p) =  .2180383387563736
               e(pbar) =  .5683930942895087
               e(xbar) =  .220061785738521
             e(offbar) =  0

macros:
                e(cmd) : "dprobit"
              e(dummy) : " 0 0 0 0 0 1 0 0 0 0 0 0"
             e(depvar) : "inlf"
           e(crittype) : "log likelihood"
            e(predict) : "probit_p"
           e(chi2type) : "LR"

matrices:
                  e(b) :  1 x 12
                  e(V) :  12 x 12
            e(se_dfdx) :  1 x 11
               e(dfdx) :  1 x 11

functions:
             e(sample)
```

```
. * Test 1:
. test dkidslt6 d6nwinc d6ed d6exp d6expsq d6age

 ( 1)  dkidslt6 = 0
 ( 2)  d6nwinc = 0
 ( 3)  d6ed = 0
 ( 4)  d6exp = 0
 ( 5)  d6expsq = 0
 ( 6)  d6age = 0

           chi2(  6) =   58.11
         Prob > chi2 =    0.0000

. return list

scalars:
              r(drop) =  0
              r(chi2) =  58.11036668348744
                r(df) =  6
                 r(p) =  1.08838734793e-10
```

```
. * Test 2:
. test d6nwinc d6ed d6exp d6expsq d6age

 ( 1)  d6nwinc = 0
 ( 2)  d6ed = 0
 ( 3)  d6exp = 0
 ( 4)  d6expsq = 0
 ( 5)  d6age = 0

           chi2(  5) =    9.03
         Prob > chi2 =    0.1078

. return list

scalars:
              r(drop) =  0
              r(chi2) =  9.031191992371875
                r(df) =  5
                 r(p) =  .1078264635420236
```

❑ Interpreting the coefficient estimates in full-interaction Model 3

Full-interaction Model 3 estimates ***two* distinct sets of probit coefficients**: (1) the probit coefficients for married women who have no pre-school aged children (for whom $dkidslt6_i = 0$); and (2) the probit coefficients for married women who have one or more pre-school aged children (for whom $dkidslt6_i = 1$).

- Recall that the **probit *index* function for Model 3** is:

$$
\begin{aligned}
x_i^T\beta = {} & \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i \\
& + \beta_6 dkidslt6_i + \beta_7 dkidslt6_i\, nwifeinc_i + \beta_8 dkidslt6_i\, ed_i \\
& + \beta_9 dkidslt6_i \exp_i + \beta_{10} dkidslt6_i \exp_i^2 + \beta_{11} dkidslt6_i\, age_i
\end{aligned}
$$

- The **probit index function for married women who have no pre-school aged children** (for whom $dkidslt6_i$ = 0) is obtained by **setting the indicator variable $dkidslt6_i = 0$** in the probit index function for Model 3:

$$\left(x_i^T\beta \,\middle|\, dkidslt6_i = 0 \right) = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i$$

Implication: The probit coefficient estimates for married women who have no pre-school aged children (for whom $dkidslt6_i = 0$) are given directly by the coefficient estimates of the first six terms in the above index function. In particular, **for married women who currently have no pre-school aged children**:

The probit coefficient estimates for married women who have no pre-school aged children are:

β_0 = the intercept coefficient for women for whom $dkidslt6_i = 0$
β_1 = the slope coefficient of $nwifeinc_i$ for women for whom $dkidslt6_i = 0$
β_2 = the slope coefficient of ed_i for women for whom $dkidslt6_i = 0$
β_3 = the slope coefficient of exp_i for women for whom $dkidslt6_i = 0$
β_4 = the slope coefficient of exp_i^2 for women for whom $dkidslt6_i = 0$
β_5 = the slope coefficient of age_i for women for whom $dkidslt6_i = 0$.

- The **probit index function for married women who currently have one or more pre-school aged children** (for whom $\text{dkidslt6}_i = 1$) is obtained by setting the indicator variable $\text{dkidslt6}_i = 1$ in the probit index function for Model 3:

$$\left(x_i^T\beta \,\middle|\, \text{dkidslt6}_i = 1\right) = \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i$$
$$+ \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i$$

Implication: The probit coefficient estimates for married women who have one or more pre-school aged children (for whom $\text{dkidslt6}_i = 1$) are obtained from Model 3 by summing pairs of coefficient estimates. In particular, **for married women who have one or more pre-school aged children**:

$\beta_0 + \beta_6 =$ the intercept coefficient for women for whom $\text{dkidslt6}_i = 1$
$\beta_1 + \beta_7 =$ the slope coefficient of nwifeinc_i for women for whom $\text{dkidslt6}_i = 1$
$\beta_2 + \beta_8 =$ the slope coefficient of ed_i for women for whom $\text{dkidslt6}_i = 1$
$\beta_3 + \beta_9 =$ the slope coefficient of $\exp_i$ for women for whom $\text{dkidslt6}_i = 1$
$\beta_4 + \beta_{10} =$ the slope coefficient of $\exp_i^2$ for women for whom $\text{dkidslt6}_i = 1$
$\beta_5 + \beta_{11} =$ the slope coefficient of age_i for women for whom $\text{dkidslt6}_i = 1$.

- Compute from Model 3 the probit coefficient estimates, t-ratios and p-values for those married women who have one or more pre-school aged children (for whom $\text{dkidslt6}_i = 1$). Enter the following **lincom** commands:

```
lincom _b[_cons] + _b[dkidslt6]
lincom _b[nwifeinc] + _b[d6nwinc]
lincom _b[ed] + _b[d6ed]
lincom _b[exp] + _b[d6exp]
lincom _b[expsq] + _b[d6expsq]
lincom _b[age] + _b[d6age]
```

```
. * Model 3 probit coefficients for women for whom dkidslt6 = 1
. lincom _b[_cons] + _b[dkidslt6]

 ( 1)  dkidslt6 + _cons = 0

------------------------------------------------------------------------------
        inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |  -1.918622   1.166582    -1.64   0.100    -4.205081    .3678365
------------------------------------------------------------------------------

. lincom _b[nwifeinc] + _b[d6nwinc]

 ( 1)  nwifeinc + d6nwinc = 0

------------------------------------------------------------------------------
        inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |  -.0168304   .0094237    -1.79   0.074    -.0353004    .0016397
------------------------------------------------------------------------------

. lincom _b[ed] + _b[d6ed]

 ( 1)  ed + d6ed = 0

------------------------------------------------------------------------------
        inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .1542988   .0556478     2.77   0.006     .0452311    .2633665
------------------------------------------------------------------------------
```

```
. lincom _b[exp] + _b[d6exp]

 ( 1)  exp + d6exp = 0

------------------------------------------------------------------------------
        inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .0244335    .062981     0.39   0.698    -.0990069    .1478739
------------------------------------------------------------------------------

. lincom _b[expsq] + _b[d6expsq]

 ( 1)  expsq + d6expsq = 0

------------------------------------------------------------------------------
        inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .0007676   .0032829     0.23   0.815    -.0056666    .0072019
------------------------------------------------------------------------------

. lincom _b[age] + _b[d6age]

 ( 1)  age + d6age = 0

------------------------------------------------------------------------------
        inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |  -.0089589   .0246402    -0.36   0.716    -.0572529     .039335
------------------------------------------------------------------------------
```

❑ Computing the marginal *probability* effect of the binary explanatory variable *dkidslt6$_i$* in Model 3 – *dprobit* with *at(vecname)* option

This section demonstrates how to use the **dprobit** command with the **at(*vecname*)** option to compute the **marginal *probability* effect of the dummy variable *dkidslt6$_i$* in Model 3** for married women who have the sample median values of the explanatory variables nwifeinc_i, ed_i, $\exp_i$, and age_i.

Here we are concerned with obtaining an estimate of the **direction and magnitude** of the **marginal *probability* effect of the dummy variable *dkidslt6$_i$* in Model 3**.

The **marginal *probability* effect of the dummy variable *dkidslt6$_i$* in Model 3** is:

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) - \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) = \Phi\left(x_{1i}^T\beta\right) - \Phi\left(x_{0i}^T\beta\right)$$

$$= \Phi\begin{pmatrix} \beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i \\ + \beta_6 + \beta_7\text{nwifeinc}_i + \beta_8\text{ed}_i + \beta_9\exp_i + \beta_{10}\exp_i^2 + \beta_{11}\text{age}_i \end{pmatrix}$$

$$- \Phi\left(\beta_0 + \beta_1\text{nwifeinc}_i + \beta_2\text{ed}_i + \beta_3\exp_i + \beta_4\exp_i^2 + \beta_5\text{age}_i\right)$$

The procedure for this computation consists of three steps.

Three-step procedure for computing the marginal *probability* effect of the dummy variable *dkidslt6$_i$* in Model 3

<u>Step 1</u>: Estimate the probability of labour force participation for married women with the specified characteristics **who currently have *one or more* dependent children under 6 years of age, for whom dkidslt6$_i$ = 1**: i.e., compute an estimate of

$$\Phi\left(x_{1i}^T\beta\right) = \Phi\begin{pmatrix}\beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i \\ + \beta_6 + \beta_7 nwifeinc_i + \beta_8 ed_i + \beta_9 \exp_i + \beta_{10} \exp_i^2 + \beta_{11} age_i\end{pmatrix}$$

<u>Step 2</u>: Estimate the probability of labour force participation for married women with the specified characteristics **who currently have *no* dependent children under 6 years of age, for whom dkidslt6$_i$ = 0**: i.e., compute an estimate of

$$\Phi\left(x_{0i}^T\beta\right) = \Phi\left(\beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i\right)$$

<u>Step 3</u>: Compute an estimate of the difference $\Phi\left(x_{1i}^T\beta\right) - \Phi\left(x_{0i}^T\beta\right)$, which is the **marginal *probability* effect of having one or more pre-school aged children** for married women who have the specified characteristics.

- Compute (or select) the values of the explanatory variables at which you wish to compute the marginal probability effect of the binary variable $dkidslt6_i$.

  Use the **pooled sample *medians*** of the explanatory variables $nwifeinc_i$, ed_i, exp_i, and age_i. Enter the following commands:

```
summarize nwifeinc, detail
return list
scalar nwinc50p = r(p50)
summarize ed, detail
scalar ed50p = r(p50)
summarize exp, detail
scalar exp50p = r(p50)
scalar expsq50p = exp50p^2
summarize age, detail
scalar age50p = r(p50)
scalar list nwinc50p ed50p exp50p expsq50p age50p
```

  The sample median values of the explanatory variables computed by these commands are as follows:

```
nwinc50p =  17.700001
   ed50p =         12
  exp50p =          9
expsq50p =         81
  age50p =         43
```

- **<u>Step 1</u>:** Use the **dprobit** command *with* the **at(*vecname*)** option to compute the marginal probability effects in Model 3 for median married women whose non-wife family income is $17,700 per year (nwifeinc = 17.700), who have 12 years of formal education (ed = 12) and 9 years of actual work experience (exp = 9, expsq = 81), who are 43 years of age (age = 43), and **who have *one or more* dependent children under 6 years of age (dkidslt6 = 1)**.

  First create the vector x_{1i}^T containing the median values of the regressors in Model 3 when $dkidslt6_i = 1$. The coefficient vector β for Model 3 in *Stata* format is:

$$\beta = \left(\beta_1 \;\; \beta_2 \;\; \beta_3 \;\; \beta_4 \;\; \beta_5 \;\; \beta_6 \;\; \beta_7 \;\; \beta_8 \;\; \beta_9 \;\; \beta_{10} \;\; \beta_{11} \;\; \beta_0\right)^T$$

  In *Stata* format, the vector x_{1i}^T for Model 3 thus takes the form:

$$x_{1i}^T = \left(nwifeinc_i \;\; ed_i \;\; exp_i \;\; exp_i^2 \;\; age_i \;\; 1 \;\; nwifeinc_i \;\; ed_i \;\; exp_i \;\; exp_i^2 \;\; age_i \;\; 1\right)$$

$$= \begin{pmatrix} nwinc50p & ed50p & exp50p & expsq50p & age50p & 1 \\ nwinc50p & ed50p & exp50p & expsq50p & age50p & 1 \end{pmatrix}$$

Step 1 *Stata* commands are:

```
matrix x1median = (nwinc50p, ed50p, exp50p, expsq50p, age50p, 1, nwinc50p, ed50p,
exp50p, expsq50p, age50p, 1)

matrix list x1median

dprobit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age,
at(x1median)

ereturn list
```

Display and save the value of $\Phi\left(x_{1i}^{T}\hat{\beta}\right)$, an estimate of $\Pr\left(\text{inlf}_i = 1 \mid \text{dkidslt6}_i = 1\right)$. The value of $\Phi\left(x_{1i}^{T}\hat{\beta}\right)$ is temporarily stored as the scalar **e(at)** following the above **dprobit** command. Enter the commands:

```
display e(at)
scalar PHIx1med = e(at)
scalar list PHIx1med
```

These commands save the value of $\Phi\left(x_{1i}^{T}\hat{\beta}\right)$ as the scalar **`PHIx1med`**.

- **<u>Step 2</u>:** Now use the **dprobit** command *with* the **at(*vecname*)** option to compute the marginal probability effects in Model 3 for median married women whose non-wife family income is $17,700 per year (nwifeinc = 17.700), who have 12 years of formal education (ed = 12) and 9 years of actual work experience (exp = 9, expsq = 81), who are 43 years of age (age = 43), and **who have *no* dependent children under 6 years of age (dkidslt6 = 0)**. Again, you will first have to create the vector x_{0i}^T containing the median values of the regressors in Model 3 when $dkidslt6_i = 0$.

  In *Stata* format, the vector x_{0i}^T for Model 3 takes the form:

$$x_{0i}^T = \left(\text{nwifeinc}_i \;\; \text{ed}_i \;\; \text{exp}_i \;\; \text{exp}_i^2 \;\; \text{age}_i \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1\right)$$

$$= \left(\text{nwinc50p} \;\; \text{ed50p} \;\; \text{exp50p} \;\; \text{expsq50p} \;\; \text{age50p} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1\right)$$

Step 2 *Stata* commands are:

```
matrix x0median = (nwinc50p, ed50p, exp50p, expsq50p, age50p, 0, 0, 0, 0, 0, 0, 1)
matrix list x0median
dprobit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age,
at(x0median)
ereturn list
```

Display and save the value of $\Phi\left(x_{0i}^{T}\hat{\beta}\right)$, an estimate of $\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right)$. The value of $\Phi\left(x_{0i}^{T}\hat{\beta}\right)$ is temporarily stored as the scalar **e(at)** following the above **dprobit** command. Enter the commands:

```
display e(at)
scalar PHIx0med = e(at)
scalar list PHIx0med
```

These commands save the value of $\Phi\left(x_{0i}^{T}\hat{\beta}\right)$ as the scalar **`PHIx0med`**.

- **<u>Step 3</u>:** Finally, compute the estimate of the difference $\Phi\left(x_{1i}^{T}\beta\right) - \Phi\left(x_{0i}^{T}\beta\right)$, which is the marginal probability effect having one or more dependent children under 6 years of age for married women who have the specified characteristics. **Step 3 *Stata* commands** are:

```
scalar diffPHImed = PHIx1med - PHIx0med
scalar list PHIx1med PHIx0med diffPHImed
```

The value of the scalar **`diffPHImed`** is **the estimate for Model 3** of

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) - \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) = \Phi\left(x_{1i}^{T}\beta\right) - \Phi\left(x_{0i}^{T}\beta\right)$$

i.e., of the **marginal *probability* effect of having one or more dependent children under 6 years of age** for married women who have the median characteristics of women in the full sample.

$$\texttt{diffPHImed} = \hat{\Pr}\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) - \hat{\Pr}\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) = \Phi\left(x_{1i}^{T}\hat{\beta}\right) - \Phi\left(x_{0i}^{T}\hat{\beta}\right)$$

Output of Step 1 *Stata* Commands

```
. dprobit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age, at(x1median)

Iteration 0:   log likelihood =  -514.8732
Iteration 1:   log likelihood = -406.48086
Iteration 2:   log likelihood = -402.63328
Iteration 3:   log likelihood = -402.61111
Iteration 4:   log likelihood = -402.61111

Probit estimates                                        Number of obs =    753
                                                        LR chi2(11)   = 224.52
                                                        Prob > chi2   = 0.0000
Log likelihood = -402.61111                             Pseudo R2     = 0.2180

------------------------------------------------------------------------------
    inlf |      dF/dx   Std. Err.      z    P>|z|        x  [    95% C.I.   ]
---------+--------------------------------------------------------------------
nwifeinc |  -.0039009   .0020603    -1.95   0.051      17.7  -.007939  .000137
      ed |   .0434699   .0113882     4.34   0.000        12   .021149   .06579
     exp |   .0490971    .009644     6.57   0.000         9   .030195  .067999
   expsq |  -.0007991   .0002526    -3.44   0.001        81  -.001294 -.000304
     age |  -.0212205   .0040365    -6.94   0.000        43  -.029132 -.013309
dkidslt6*|  -.6603895   .0730752    -1.99   0.046         1  -.803614 -.517165
 d6nwinc |  -.0021167   .0039297    -0.54   0.589      17.7  -.009819  .005585
    d6ed |    .011699   .0221757     0.53   0.600        12  -.031765  .055162
   d6exp |   -.040361   .0215344    -1.70   0.089         9  -.082568  .001846
 d6expsq |   .0010736   .0011221     0.90   0.370        81  -.001126  .003273
   d6age |   .0180172   .0111044     1.93   0.053        43  -.003747  .039781
---------+--------------------------------------------------------------------
  obs. P |   .5683931
 pred. P |   .5870885  (at x-bar)
 pred. P |   .3198606  (at x)
------------------------------------------------------------------------------
(*) dF/dx is for discrete change of dummy variable from 0 to 1
    z and P>|z| are the test of the underlying coefficient being 0
```

```
. ereturn list

scalars:
                  e(N) =  753
               e(ll_0) =  -514.8732045671461
                 e(ll) =  -402.6111063731551
               e(df_m) =  11
               e(chi2) =  224.5241963879821
               e(r2_p) =  .2180383387563736
               e(pbar) =  .5683930942895087
               e(xbar) =  .220061785738521
             e(offbar) =  0
                 e(at) =  .3198606279066483

[output omitted]

. display e(at)
.31986063

. scalar PHIx1med = e(at)

. scalar list PHIx1med
  PHIx1med =  .31986063
```

Output of Step 2 *Stata* Commands

```
. dprobit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age, at(x0median);

Iteration 0:   log likelihood =  -514.8732
Iteration 1:   log likelihood = -406.48086
Iteration 2:   log likelihood = -402.63328
Iteration 3:   log likelihood = -402.61111
Iteration 4:   log likelihood = -402.61111

Probit estimates                                        Number of obs =    753
                                                        LR chi2(11)   = 224.52
                                                        Prob > chi2   = 0.0000
Log likelihood = -402.61111                             Pseudo R2     = 0.2180

------------------------------------------------------------------------------
    inlf |      dF/dx   Std. Err.      z    P>|z|         x  [    95% C.I.   ]
---------+--------------------------------------------------------------------
nwifeinc |   -.004054   .0020554    -1.95   0.051      17.7  -.008083 -.000025
      ed |   .0451757   .0104184     4.34   0.000        12   .024756  .065595
     exp |   .0510237   .0074085     6.57   0.000         9   .036503  .065544
   expsq |  -.0008305   .0002325    -3.44   0.001        81  -.001286 -.000375
     age |  -.0220532    .003204    -6.94   0.000        43  -.028333 -.015773
dkidslt6*|  -.6311359   .0559456    -1.99   0.046         0  -.740787 -.521485
 d6nwinc |  -.0021998   .0040816    -0.54   0.589         0  -.010199    .0058
    d6ed |    .012158   .0231649     0.53   0.600         0  -.033244   .05756
   d6exp |  -.0419448   .0245612    -1.70   0.089         0  -.090084  .006194
 d6expsq |   .0011157   .0012413     0.90   0.370         0  -.001317  .003549
   d6age |   .0187242   .0096966     1.93   0.053         0  -.000281  .037729
---------+--------------------------------------------------------------------
  obs. P |   .5683931
 pred. P |   .5870885  (at x-bar)
 pred. P |   .6469122  (at x)
------------------------------------------------------------------------------
(*) dF/dx is for discrete change of dummy variable from 0 to 1
    z and P>|z| are the test of the underlying coefficient being 0
```

```
. ereturn list

scalars:
                  e(N) =  753
               e(ll_0) =  -514.8732045671461
                 e(ll) =  -402.6111063731551
               e(df_m) =  11
               e(chi2) =  224.5241963879821
               e(r2_p) =  .2180383387563736
               e(pbar) =  .5683930942895087
               e(xbar) =  .220061785738521
             e(offbar) =  0
                 e(at) =  .6469121653332525

[output omitted]

. display e(at)
.64691217

. scalar PHIx0med = e(at)

. scalar list PHIx0med
  PHIx0med =  .64691217
```

Output of Step 3 *Stata* Commands

```
.
. * Model 3: compute marginal probability effect of dkidslt6
. scalar diffPHImed = PHIx1med - PHIx0med

. scalar list PHIx1med PHIx0med diffPHImed
  PHIx1med =  .31986063
  PHIx0med =  .64691217
diffPHImed = -.32705154
```

The value of the scalar **`diffPHImed`** is **the estimate for Model 3** of

$$\Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 1\right) - \Pr\left(\text{inlf}_i = 1 \middle| \text{dkidslt6}_i = 0\right) = \Phi\left(x_{1i}^T\beta\right) - \Phi\left(x_{0i}^T\beta\right)$$

In Model 3, the estimated **marginal *probability* effect of having one or more dependent children under 6 years of age** for married women who have the median characteristics of women in the full sample is:

$$\Phi\left(x_{1i}^T\hat{\beta}\right) - \Phi\left(x_{0i}^T\hat{\beta}\right) = \mathbf{-0.32705154} = \mathbf{-0.3271}$$

❑ Marginal *probability* effects of *continuous* explanatory variables in Model 3 -- *dprobit*

Background

- The **marginal *probability* effects of *continuous* explanatory variables in probit models** are the partial derivatives of the standard normal c.d.f. $\Phi\left(x_i^T\beta\right)$ with respect to the individual explanatory variables:

marginal *probability* effect of $\mathbf{X_j}$ $= \dfrac{\partial\,\Phi\left(x_i^T\beta\right)}{\partial\,X_{ij}} = \dfrac{\partial\,\Phi\left(x_i^T\beta\right)}{\partial\,x_i^T\beta}\dfrac{\partial\,x_i^T\beta}{\partial\,X_{ij}} = \phi\left(x_i^T\beta\right)\dfrac{\partial\,x_i^T\beta}{\partial\,X_{ij}}$

where

$\phi\left(x_i^T\beta\right)$ = the **value of the standard normal p.d.f.** evaluated at $x_i^T\beta$

$\dfrac{\partial\,x_i^T\beta}{\partial\,X_{ij}}$ = the **marginal *index* effect** of the continuous variable X_j.

- Recall that the **probit index function for Model 3** is:

$$
\begin{aligned}
x_i^T\beta \;=\; & \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i \\
& + \beta_6 dkidslt6_i + \beta_7 dkidslt6_i\,nwifeinc_i + \beta_8 dkidslt6_i\,ed_i \\
& + \beta_9 dkidslt6_i \exp_i + \beta_{10} dkidslt6_i \exp_i^2 + \beta_{11} dkidslt6_i\,age_i
\end{aligned}
$$

Marginal *Index* Effects of Continuous Explanatory Variables – Model 3

- For Model 3, there are ***two* sets of marginal index effects**, one for women with no pre-school aged children (for whom $dkidslt6_i = 0$), and the other for women with one or more pre-school aged children (for whom $dkidslt6_i = 1$).

- The **marginal *index* effects** of the ***continuous* explanatory variables in Model 3** are obtained by partially differentiating the index function $x_i^T\beta$ for Model 3 with respect to each of the four continuous explanatory variables $nwifeinc_i$, ed_i, exp_i, and age_i.

  The **probit index function**, or regression function, **for Model 3** is:

$$\begin{aligned} x_i^T\beta = {} & \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i \\ & + \beta_6 dkidslt6_i + \beta_7 dkidslt6_i\, nwifeinc_i + \beta_8 dkidslt6_i\, ed_i \\ & + \beta_9 dkidslt6_i \exp_i + \beta_{10} dkidslt6_i \exp_i^2 + \beta_{11} dkidslt6_i\, age_i \end{aligned}$$

$$x_i^T\beta = \beta_0 + \beta_1 nwifeinc_i + \beta_2 ed_i + \beta_3 \exp_i + \beta_4 \exp_i^2 + \beta_5 age_i$$
$$+ \beta_6 dkidslt6_i + \beta_7 dkidslt6_i\, nwifeinc_i + \beta_8 dkidslt6_i\, ed_i$$
$$+ \beta_9 dkidslt6_i \exp_i + \beta_{10} dkidslt6_i \exp_i^2 + \beta_{11} dkidslt6_i\, age_i$$

Now partially differentiate the index function $x_i^T\beta$ for Model 3 with respect to each of the four continuous explanatory variables $nwifeinc_i$, ed_i, $\exp_i$, and age_i.

1. marginal index effect of $nwifeinc_i$ = $\dfrac{\partial x_i^T\beta}{\partial nwifeinc_i} = \beta_1 + \beta_7 dkidslt6_i$

2. marginal index effect of ed_i = $\dfrac{\partial x_i^T\beta}{\partial ed_i} = \beta_2 + \beta_8 dkidslt6_i$

3. marginal index effect of $\exp_i$ = $\dfrac{\partial x_i^T\beta}{\partial \exp_i}$

   $= \beta_3 + 2\beta_4 \exp_i + (\beta_9 + 2\beta_{10} \exp_i) dkidslt6_i$

4. marginal index effect of age_i = $\dfrac{\partial x_i^T\beta}{\partial age_i} = \beta_5 + \beta_{11} dkidslt6_i$

Note: Each of these marginal *index* effects differs depending on whether $dkidslt6_i = 0$ or $dkidslt6_i = 1$.

- The **marginal index effects for married women with *no pre-school aged children*** are obtained by **setting the indicator variable $\mathbf{dkidslt6_i = 0}$** in expressions 1 to 4 above:

5. marginal index effect of nwifeinc_i = $\dfrac{\partial x_i^T\beta}{\partial\, \text{nwifeinc}_i} = \beta_1$

6. marginal index effect of ed_i = $\dfrac{\partial x_i^T\beta}{\partial\, \text{ed}_i} = \beta_2$

7. marginal index effect of exp_i = $\dfrac{\partial x_i^T\beta}{\partial\, \text{exp}_i} = \beta_3 + 2\beta_4\, \text{exp}_i$

8. marginal index effect of age_i = $\dfrac{\partial x_i^T\beta}{\partial\, \text{age}_i} = \beta_5$

- The **marginal index effects for married women with *one or more pre-school aged children*** are obtained by **setting the indicator variable $dkidslt6_i = 1$** in expressions 1 to 4 above:

9. marginal index effect of $nwifeinc_i$ = $\dfrac{\partial x_i^T\beta}{\partial nwifeinc_i} = \beta_1 + \beta_7$

10. marginal index effect of ed_i = $\dfrac{\partial x_i^T\beta}{\partial ed_i} = \beta_2 + \beta_8$

11. marginal index effect of exp_i = $\dfrac{\partial x_i^T\beta}{\partial exp_i} = \beta_3 + 2\beta_4 \exp_i + (\beta_9 + 2\beta_{10} \exp_i)$

$$= \beta_3 + \beta_9 + 2(\beta_4 + \beta_{10})\exp_i$$

12. marginal index effect of age_i = $\dfrac{\partial x_i^T\beta}{\partial age_i} = \beta_5 + \beta_{11}$

Marginal *Probability* Effects of Continuous Explanatory Variables – Model 3

- The **marginal *probability* effects** of the four continuous explanatory variables in Model 3 are:

1. marginal probability effect of nwifeinc_i $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial \,\text{nwifeinc}_i} = \phi(x_i^T\beta)\dfrac{\partial x_i^T\beta}{\partial \,\text{nwifeinc}_i}$

$$= \phi(x_i^T\beta)(\beta_1 + \beta_7 \text{dkidslt6}_i)$$

2. marginal probability effect of ed_i $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial \,\text{ed}_i} = \phi(x_i^T\beta)\dfrac{\partial x_i^T\beta}{\partial \,\text{ed}_i}$

$$= \phi(x_i^T\beta)(\beta_2 + \beta_8 \text{dkidslt6}_i)$$

3. marginal probability effect of exp_i $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial \,\text{exp}_i} = \phi(x_i^T\beta)\dfrac{\partial x_i^T\beta}{\partial \,\text{exp}_i}$

$$= \phi(x_i^T\beta)(\beta_3 + 2\beta_4 \,\text{exp}_i + (\beta_9 + 2\beta_{10}\,\text{exp}_i)\text{dkidslt6}_i)$$

4. marginal probability effect of age_i $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial \,\text{age}_i} = \phi(x_i^T\beta)\dfrac{\partial x_i^T\beta}{\partial \,\text{age}_i}$

$$= \phi(x_i^T\beta)(\beta_5 + \beta_{11} \text{dkidslt6}_i)$$

Notes: There are **three features of these marginal probability effects for Model 3** that you should recognize.

1. These marginal *probability* effects differ depending on whether $\text{dkidslt6}_i = 0$ or $\text{dkidslt6}_i = 1$.

2. The marginal probability effect of a continuous explanatory variable X_j is proportional to the marginal index effect of X_j, where the factor of proportionality is the standard normal p.d.f. at $x_i^T\beta$:

   marginal *probability* effect of X_j $= \phi\left(x_i^T\beta\right) \times$ **marginal *index* effect of X_j**

3. Estimation of the marginal probability effects of a continuous explanatory variable X_j requires one to choose a specific vector of regressor values x_i^T. Common choices for x_i^T are the **sample *mean*** and **sample *median*** values of the regressors.

- The **marginal *probability* effects for married women with *no pre-school aged children*** are obtained by setting the indicator variable **$dkidslt6_i = 0$** in expressions 1 to 4 above:

5. marginal probability effect of $nwifeinc_i$ $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial\, nwifeinc_i} = \phi(x_i^T\beta)\dfrac{\partial\, x_i^T\beta}{\partial\, nwifeinc_i} = \phi(x_i^T\beta)\beta_1$

6. marginal probability effect of ed_i $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial\, ed_i} = \phi(x_i^T\beta)\dfrac{\partial\, x_i^T\beta}{\partial\, ed_i} = \phi(x_i^T\beta)\beta_2$

7. marginal probability effect of exp_i $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial\, exp_i} = \phi(x_i^T\beta)\dfrac{\partial\, x_i^T\beta}{\partial\, exp_i} = \phi(x_i^T\beta)(\beta_3 + 2\beta_4\, exp_i)$

8. marginal probability effect of age_i $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial\, age_i} = \phi(x_i^T\beta)\dfrac{\partial\, x_i^T\beta}{\partial\, age_i} = \phi(x_i^T\beta)\beta_5$

- The **marginal *probability* effects for married women with *one or more pre-school aged children*** are obtained by setting the indicator variable **dkidslt6$_i$ = 1** in expressions 1 to 4 above:

9. marginal probability effect of nwifeinc$_i$ $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial\, \text{nwifeinc}_i} = \phi(x_i^T\beta)\dfrac{\partial\, x_i^T\beta}{\partial\, \text{nwifeinc}_i}$

$$= \phi(x_i^T\beta)(\beta_1 + \beta_7)$$

10. marginal probability effect of ed$_i$ $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial\, \text{ed}_i} = \phi(x_i^T\beta)\dfrac{\partial\, x_i^T\beta}{\partial\, \text{ed}_i}$

$$= \phi(x_i^T\beta)(\beta_2 + \beta_8)$$

11. marginal probability effect of exp$_i$ $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial\, \text{exp}_i} = \phi(x_i^T\beta)\dfrac{\partial\, x_i^T\beta}{\partial\, \text{exp}_i}$

$$= \phi(x_i^T\beta)(\beta_3 + 2\beta_4\,\text{exp}_i + \beta_9 + 2\beta_{10}\,\text{exp}_i)$$

$$= \phi(x_i^T\beta)(\beta_3 + \beta_9 + 2(\beta_4 + \beta_{10})\,\text{exp}_i)$$

12. marginal probability effect of age$_i$ $= \dfrac{\partial \Phi(x_i^T\beta)}{\partial\, \text{age}_i} = \phi(x_i^T\beta)\dfrac{\partial\, x_i^T\beta}{\partial\, \text{age}_i}$

$$= \phi(x_i^T\beta)(\beta_5 + \beta_{11})$$

❑ Testing for zero marginal *probability* effects of *continuous* explanatory variables in Model 3 – *probit* or *dprobit*

<u>Background</u>:

For any explanatory variable, there are ***two*** **distinct empirical questions** that an econometric investigation of married women's labour force participation (or any other binary outcome) should address.

- The first question concerns the ***existence*** **of a relationship**: is a particular explanatory variable related to the probability of married women's labour force participation, conditional on other explanatory variables included in the model?

  In other words, is the marginal probability effect of a particular explanatory variable on the probability of married women's labour force participation zero or non-zero?

- The second question concerns the ***direction*** **and** ***magnitude*** **of the relationship**: how large a change in the conditional probability of married women's labour force participation is associated with a one-unit increase in the value of a particular continuous explanatory variable, holding constant the values of all other explanatory variables included in the model?

This section addresses the first question for each of the four continuous variables in Model 3.

Objective: To test the proposition that the **marginal effect of each *continuous* explanatory variable** on the probability of married women's labour force participation **is equal to *zero*** for each of the ***two* groups of married women**:

1. married women with one or more pre-school aged children
and
2. married women with no pre-school aged children

Important Point:

The marginal probability effect of a continuous explanatory variable X_j is proportional to the marginal index effect of X_j, where the factor of proportionality is the standard normal p.d.f. at $x_i^T\beta$:

marginal *probability* effect of X_j $= \phi\left(x_i^T\beta\right) \times$ **marginal *index* effect of X_j**

Implication: Any set of coefficient restrictions that is sufficient to make the **marginal *index* effect** of a continuous explanatory variable **equal to zero** is also sufficient to make the **marginal *probability* effect** of that continuous explanatory variable **equal to zero**.

In other words, testing the null hypothesis that the marginal *index* effect of a continuous explanatory variable equals zero is equivalent to testing the null hypothesis that the marginal *probability* effect of that continuous explanatory variable equals zero.

- First, re-estimate probit Model 3. Enter the **probit** command:

```
probit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age
```

- ***<u>Test 1 - Model 3</u>: for married women with no pre-school aged children***

- ***Proposition:*** The non-wife income of the family has no effect on the probability of labour force participation for married women who have no pre-school aged children; the **marginal *probability* (and *index*) effect of $\mathbf{nwifeinc_i}$ equals zero** for married women for whom $dkidslt6_i = 0$.

- ***For married women for whom $dkidslt6_i = 0$:*** marginal probability effect of $nwifeinc_i = \phi(x_i^T\beta)\beta_1$

  A sufficient condition for the marginal probability effect of $nwifeinc_i$ to equal zero for any given values of the regressors x_i^T is $\beta_1 = 0$.

- ***Null and Alternative Hypotheses:***

  H_0: $\beta_1 = 0$ versus H_1: $\beta_1 \neq 0$

- To calculate a **Wald test** of this hypothesis and the corresponding p-value for the calculated W-statistic, enter the following **test**, **return list** and **display** commands:

```
test nwifeinc     or   test nwifeinc = 0
return list
display sqrt(r(chi2))
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following *Stata* commands:

```
lincom _b[nwifeinc]
return list
display r(estimate)/r(se)
```

  The results of this two-tail t-test are identical with those of the previous Wald test.

- ***Test 1 - Model 3*: for married women with one or more pre-school aged children**

- ***Proposition:*** The non-wife income of the family has no effect on the probability of labour force participation for married women who have one or more pre-school aged children; the **marginal *probability* (and *index*) effect of $\mathbf{nwifeinc_i}$ equals zero** for married women for whom $dkidslt6_i = 1$.

- ***For married women for whom $dkidslt6_i = 1$:*** marginal probability effect of $nwifeinc_i = \phi(x_i^T\beta)(\beta_1 + \beta_7)$

  A sufficient condition for the marginal probability effect of $nwifeinc_i$ to equal zero for any given values of the regressors x_i^T is $\beta_1 + \beta_7 = 0$.

- ***Null and Alternative Hypotheses:***

  H_0: $\beta_1 + \beta_7 = 0$ versus H_1: $\beta_1 + \beta_7 \neq 0$

- To calculate a **Wald test** of this hypothesis and the corresponding p-value for the calculated W-statistic, enter the following **test**, **return list** and **display** commands:

```
test nwifeinc + d6nwinc = 0
return list
display sqrt(r(chi2))
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following *Stata* commands:

```
lincom _b[nwifeinc] + _b[d6nwinc]
return list
display r(estimate)/r(se)
```

  The results of this two-tail t-test are identical with those of the previous Wald test.

- ***<u>Test 2 - Model 3</u>: for married women with no pre-school aged children***

- ***Proposition:*** For married women who have no pre-school aged children, the probability of labour force participation does not depend on their education; the **marginal *probability* (and *index*) effect of ed_i equals zero** for married women for whom $dkidslt6_i = 0$.

- ***For married women for whom $dkidslt6_i = 0$:*** marginal probability effect of $ed_i = \phi(x_i^T\beta)\beta_2$

  A sufficient condition for the marginal probability effect of ed_i to equal zero for any given values of the regressors x_i^T is $\beta_2 = 0$.

- ***Null and Alternative Hypotheses:***

  H_0: $\beta_2 = 0$ versus H_1: $\beta_2 \neq 0$

- To calculate a **Wald test** of this hypothesis and the corresponding p-value for the calculated W-statistic, enter the following **test**, **return list** and **display** commands:

```
test ed      or      test ed = 0
return list
display sqrt(r(chi2))
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following *Stata* commands:

```
lincom _b[ed]
return list
display r(estimate)/r(se)
```

  The results of this two-tail t-test are identical with those of the previous Wald test.

- ***<u>Test 2 - Model 3</u>: for married women with one or more pre-school aged children***

- ***Proposition:*** For married women who have one or more pre-school aged children, the probability of labour force participation does not depend on their education; the **marginal *probability* (and *index*) effect of ed_i equals zero** for married women for whom $dkidslt6_i = 1$.

- ***For married women for whom $dkidslt6_i = 1$:*** marginal probability effect of $ed_i = \phi(x_i^T\beta)(\beta_2 + \beta_8)$

  A sufficient condition for the marginal probability effect of ed_i to equal zero for any given values of the regressors x_i^T is $\beta_2 + \beta_8 = 0$.

- ***Null and Alternative Hypotheses:***

  H_0: $\beta_2 + \beta_8 = 0$ versus H_1: $\beta_2 + \beta_8 \neq 0$

- To calculate a **Wald test** of this hypothesis and the corresponding p-value for the calculated W-statistic, enter the following **test**, **return list** and **display** commands:

```
test ed + d6ed = 0
return list
display sqrt(r(chi2))
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following *Stata* commands:

```
lincom _b[ed] + _b[d6ed]
return list
display r(estimate)/r(se)
```

  The results of this two-tail t-test are identical with those of the previous Wald test.

- ***<u>Test 3 - Model 3</u>: for married women with no pre-school aged children***

- ***Proposition:*** Years of actual work experience have no effect on the probability of labour force participation for married women who have no pre-school aged children; the **marginal *probability* (and *index*) effect of $\mathbf{exp_i}$ equals zero** for married women for whom $dkidslt6_i = 0$.

- ***For married women for whom $dkidslt6_i = 0$:*** marginal probability effect of $exp_i \;=\; \phi\left(x_i^T\beta\right)\left(\beta_3 + 2\beta_4\, exp_i\right)$

  A sufficient condition for the marginal probability effect of exp_i to equal zero for any given values of the regressors x_i^T is $\beta_3 = 0$ and $\beta_4 = 0$.

- ***Null and Alternative Hypotheses:***

  H_0: $\beta_3 = 0$ *and* $\beta_4 = 0$ versus H_1: $\beta_3 \neq 0$ *and/or* $\beta_4 \neq 0$

- To calculate a **Wald test** of this hypothesis and the corresponding p-value for the calculated W-statistic, enter the following **test**, **return list** and **display** commands:

```
test exp expsq
return list
```

- ***<u>Test 3 - Model 3</u>: for married women with one or more pre-school aged children***

- ***Proposition:*** Years of actual work experience have no effect on the probability of labour force participation for married women who have one or more pre-school aged children; the **marginal *probability* (and *index*) effect of $\mathbf{exp_i}$ equals zero** for married women for whom $\text{dkidslt6}_i = 1$.

- ***For married women for whom $dkidslt6_i = 1$:***

$$\text{marginal probability effect of } \exp_i \;=\; \phi\left(x_i^T\beta\right)\left(\beta_3 + \beta_9 + 2(\beta_4 + \beta_{10})\exp_i\right)$$

A sufficient condition for the marginal probability effect of $\exp_i$ to equal zero for any given values of the regressors x_i^T is $\beta_3 + \beta_9 = 0$ and $\beta_4 + \beta_{10} = 0$.

- ***Null and Alternative Hypotheses:***

H_0: $\beta_3 + \beta_9 = 0$ *and* $\beta_4 + \beta_{10} = 0$
H_1: $\beta_3 + \beta_9 \neq 0$ *and/or* $\beta_4 + \beta_{10} \neq 0$

- To calculate a **Wald test** of this hypothesis and the corresponding p-value for the calculated W-statistic, enter the following **test** and **return list** commands:

```
test exp + d6exp = 0, notest
test expsq + d6expsq = 0, accumulate
return list
```

- ***<u>Test 4 - Model 3</u>: for married women with no pre-school aged children***

- ***Proposition:*** For married women who have no pre-school aged children, their age has no effect on their probability of labour force participation; the **marginal *probability* (and *index*) effect of age$_i$ equals zero** for married women for whom dkidslt6$_i$ = 0.

- ***For married women for whom dkidslt6$_i$ = 0:*** marginal probability effect of age$_i$ $= \phi(x_i^T\beta)\beta_5$

  A sufficient condition for the marginal probability effect of ed$_i$ to equal zero for any given values of the regressors x_i^T is $\beta_5 = 0$.

- ***Null and Alternative Hypotheses:***

  H_0: $\beta_5 = 0$ versus H_1: $\beta_5 \neq 0$

- To calculate a **Wald test** of this hypothesis and the corresponding p-value for the calculated W-statistic, enter the following **test**, **return list** and **display** commands:

```
test age        or      test age = 0
return list
display sqrt(r(chi2))
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following *Stata* commands:

```
lincom _b[age]
return list
display r(estimate)/r(se)
```

  The results of this two-tail t-test are identical with those of the previous Wald test.

- ***<u>Test 4 - Model 3</u>: for married women with one or more pre-school aged children***

- ***Proposition:*** For married women who have one or more pre-school aged children, their age has no effect on their probability of labour force participation; the **marginal *probability* (and *index*) effect of age$_i$ equals zero** for married women for whom dkidslt6$_i$ = 1.

- ***For married women for whom dkidslt6$_i$ = 1:*** marginal probability effect of age$_i$ $= \phi\left(x_i^T\beta\right)\left(\beta_5 + \beta_{11}\right)$

  A sufficient condition for the marginal probability effect of age$_i$ to equal zero for any given values of the regressors x_i^T is $\beta_5 + \beta_{11} = 0$.

- ***Null and Alternative Hypotheses:***

  H_0: $\beta_5 + \beta_{11} = 0$ versus H_1: $\beta_5 + \beta_{11} \neq 0$

- To calculate a **Wald test** of this hypothesis and the corresponding p-value for the calculated W-statistic, enter the following **test**, **return list** and **display** commands:

```
test age + d6age = 0
return list
display sqrt(r(chi2))
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following *Stata* commands:

```
lincom _b[age] + _b[d6age]
return list
display r(estimate)/r(se)
```

  The results of this two-tail t-test are identical with those of the previous Wald test.

❑ Testing for *differences* in the marginal *probability* effects of *continuous* explanatory variables in Model 3 – *probit* or *dprobit*

Objective: To test the proposition that the **marginal effect of each *continuous* explanatory variable** on the probability of married women's labour force participation **is equal** for the ***two* groups of married women**: married women with no pre-school aged children, for whom $dkidslt6_i = 0$; and married women with one or more pre-school aged children, for whom $dkidslt6_i = 1$.

- ***Test 5 - Model 3: equal* marginal *probability* effects of *nwifeinc$_i$***

- ***Proposition:*** The marginal probability (and index) effect of nwifeinc$_i$ is equal for zero for married women for whom dkidslt6$_i$ = 1.

- **Marginal *probability* effects for *nwifeinc$_i$*** are:

$$= \phi\left(x_i^T\beta\right)\beta_1 \qquad \text{when dkidslt6}_i = 0$$

$$= \phi\left(x_i^T\beta\right)\left(\beta_1 + \beta_7\right) \qquad \text{when dkidslt6}_i = 1$$

A sufficient condition for the marginal probability effect of nwifeinc$_i$ to be equal for married women with and without pre-school aged children is $\beta_7 = 0$.

- ***Null and Alternative Hypotheses:***

H_0: $\beta_7 = 0$ versus H_1: $\beta_7 \neq 0$

- To calculate a **Wald test** of this hypothesis, enter the following **test** command:

```
test d6nwinc = 0
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following **lincom** command:

```
lincom _b[d6nwinc]
```

The results of this two-tail t-test are identical with those of the previous Wald test.

- ***Test 6 - Model 3*: *equal* marginal *probability* effects of *ed***$_i$

- ***Proposition:*** The marginal probability (and index) effect of ed_i is equal for married women with pre-school aged kids and married women with no pre-school aged kids.

- **Marginal *probability* effects for *ed***$_i$ are:

$$= \phi\left(x_i^T\beta\right)\beta_2 \qquad \text{when } dkidslt6_i = 0$$

$$= \phi\left(x_i^T\beta\right)\left(\beta_2 + \beta_8\right) \qquad \text{when } dkidslt6_i = 1$$

A sufficient condition for the marginal probability effect of ed_i to be equal for married women with and without pre-school aged children is $\beta_8 = 0$.

- ***Null and Alternative Hypotheses:***

H_0: $\beta_8 = 0$ versus H_1: $\beta_8 \neq 0$

- To calculate a **Wald test** of this hypothesis, enter the following **test** command:

```
test d6ed = 0
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following **lincom** command:

```
lincom _b[d6ed]
```

The results of this two-tail t-test are identical with those of the previous Wald test.

- ***Test 7 - Model 3*: *equal* marginal *probability* effects of exp_i**

- ***Proposition:*** The marginal probability (and index) effect of exp_i is equal for married women with pre-school aged kids and married women with no pre-school aged kids.

- **Marginal *probability* effects for exp_i** are:

$$= \phi(x_i^T\beta)(\beta_3 + 2\beta_4 \exp_i) \qquad \text{when dkidslt6}_i = 0$$

$$= \phi(x_i^T\beta)(\beta_3 + \beta_9 + 2(\beta_4 + \beta_{10})\exp_i) \qquad \text{when dkidslt6}_i = 1$$

Sufficient conditions for the marginal probability effect of exp_i to be equal for married women with and without pre-school aged children are $\beta_9 = 0$ and $\beta_{10} = 0$.

- ***Null and Alternative Hypotheses:***

H_0: $\beta_9 = 0$ *and* $\beta_{10} = 0$ versus H_1: $\beta_9 \neq 0$ *and/or* $\beta_{10} \neq 0$

- To calculate a **Wald test** of this hypothesis, enter the following **test** commands:

```
test d6exp = 0
test d6expsq = 0, accumulate
```

- ***Test 8 - Model 3*: *equal* marginal *probability* effects of *age$_i$***

- ***Proposition:*** The marginal probability (and index) effect of age$_i$ is equal for married women with pre-school aged kids and married women with no pre-school aged kids.

- **Marginal *probability* effects for *age$_i$*** are:

$$= \phi(x_i^T\beta)\beta_5 \qquad \text{when dkidslt6}_i = 0$$

$$= \phi(x_i^T\beta)(\beta_5 + \beta_{11}) \qquad \text{when dkidslt6}_i = 1$$

A sufficient condition for the marginal probability effect of age$_i$ to be equal for married women with and without pre-school aged children is $\beta_{11} = 0$.

- ***Null and Alternative Hypotheses:***

$$H_0: \beta_{11} = 0 \qquad \text{versus} \qquad H_1: \beta_{11} \neq 0$$

- To calculate a **Wald test** of this hypothesis, enter the following **test** command:

```
test d6age = 0
```

- To calculate a **two-tail asymptotic t-test** of H_0 against H_1, enter the following **lincom** command:

```
lincom _b[d6age]
```

The results of this two-tail t-test are identical with those of the previous Wald test.

❑ **Computing estimates of the marginal *probability* effects of *continuous* explanatory variables in Model 3 -- *dprobit***

Objective

To estimate the ***magnitude*** of the relationship between a continuous explanatory variable and the conditional probability of married women's labour force participation.

***Question addressed is*: How large a change** in the **conditional probability of married women's labour force participation** is associated with a **one-unit *increase* in the value of a particular continuous explanatory variable**, holding constant the values of all other explanatory variables included in the model?

This section demonstrates how to address this second question for each of the continuous explanatory variables nwifeinc_i, ed_i, exp_i, and age_i.

Procedure

Recall that the **marginal *probability* effect of a *continuous* explanatory variable X_j** is **proportional to the marginal *index* effect of X_j**, where the factor of proportionality is the standard normal p.d.f. evaluated at $x_i^T\beta$:

marginal *probability* effect of X_j $= \phi\left(x_i^T\beta\right) \times$ **marginal *index* effect of X_j**

This expression implies that to compute estimates of the marginal *probability* effect of each *continuous* explanatory variable, we must first do two things.

- First, we must compute an estimate $x_i^T\hat{\beta}$ of $x_i^T\beta$.
- Second, we must compute the value of $\phi\left(x_i^T\hat{\beta}\right)$, i.e., the value of the standard normal density function evaluated at $x_i^T\hat{\beta}$.

Which *Stata* command to use Use the **dprobit** command with the **at(*vecname*)** option

Marginal *probability* effects for married women for whom *dkidslt6$_i$ = 0*

Compute the marginal probability effects of the four ***continuous* explanatory variables** in Model 3 for married women who have the **sample *median* values** of $nwifeinc_i$, ed_i, exp_i, and age_i, and **no pre-schooled aged children** (for whom $dkidslt6_i = 0$).

- First re-estimate Model 3 using the **dprobit** command with the **at(*vecname*)** option. The vector to use in the **at(*vecname*)** option is the vector x_{0i}^T containing the median values of the regressors in Model 3 when $dkidslt6_i = 0$:

$$x_{0i}^T = \left(nwifeinc_i \;\; ed_i \;\; exp_i \;\; exp_i^2 \;\; age_i \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1\right)$$
$$= \left(nwinc50p \;\; ed50p \;\; exp50p \;\; expsq50p \;\; age50p \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1\right)$$

You previously created the vector x_{0i}^T and named it **x0median**. So simply enter the commands:

```
dprobit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age,
at(x0median)
ereturn list
display e(at)
```

Recall that the scalar **e(at)** contains the value of $\Phi\left(x_{0i}^T\hat{\beta}\right)$ generated by the previous **dprobit** command, where $\Phi\left(x_{0i}^T\hat{\beta}\right)$ is an estimate of $\Pr\left(inlf_i = 1 \mid dkidslt6_i = 0\right)$.

- Second, use the *Stata* statistical function **invnormal( )** to save the value of $x_{0i}^{T}\hat{\beta}$. Enter the commands:

```
scalar x0medbhat = invnormal(e(at))
scalar list x0medbhat
```

- Third, use the *Stata* statistical function **normalden( )** to save as a scalar the value of $\phi\left(x_{0i}^{T}\hat{\beta}\right)$, which is the standard normal density function (or p.d.f.) evaluated at $x_{0i}^{T}\hat{\beta}$. Enter the commands:

```
scalar phix0med = normalden(x0medbhat)
scalar list phix0med
```

These commands save the value of $\phi\left(x_{0i}^{T}\hat{\beta}\right)$ as the scalar **phix0med**.

- Compute the **estimated marginal *probability* effect of explanatory variable *nwifeinc$_i$*** for the ***median* married woman who has *no pre-school aged children***, which when **dkidslt6$_i$ = 0** is given by the function:

***estimated* marginal *probability* effect of nwifeinc$_i$** $= \phi\left(x_{0i}^{T}\hat{\beta}\right)\hat{\beta}_1$

Enter the **lincom** command:

```
lincom phix0med*_b[nwifeinc]
```

- Compute the **estimated marginal *probability* effect of explanatory variable *ed_i*** for the ***median* married woman who has *no pre-school aged children***, which when **$dkidslt6_i = 0$** is given by the function:

  ***estimated* marginal *probability* effect of ed_i** $= \phi\left(x_{0i}^T\hat{\beta}\right)\hat{\beta}_2$

  Enter the **lincom** command:

```
lincom phix0med*_b[ed]
```

- Compute the **estimated marginal *probability* effect of explanatory variable *exp_i*** for the ***median* married woman who has *no pre-school aged children***, which when **$dkidslt6_i = 0$** is given by the function:

  ***estimated* marginal *probability* effect of exp_i** $= \phi\left(x_{0i}^T\hat{\beta}\right)\left(\hat{\beta}_3 + 2\hat{\beta}_4\, exp50p\right)$

  Enter the **lincom** command:

```
lincom phix0med*(_b[exp] + 2*_b[expsq]*exp50p)
```

- Compute the **estimated marginal *probability* effect of explanatory variable *age_i*** for the ***median* married woman who has *no pre-school aged children***, which when **$dkidslt6_i = 0$** is given by the function:

  ***estimated* marginal *probability* effect of age_i** $= \phi\left(x_{0i}^T\hat{\beta}\right)\hat{\beta}_5$

  Enter the **lincom** command:

```
lincom phix0med*_b[age]
```

Marginal *probability* effects for married women for whom *dkidslt6$_i$ = 1*

Compute the marginal probability effects of the four ***continuous* explanatory variables** in Model 3 for married women who have the **sample *median* values** of nwifeinc_i, ed_i, exp_i, and age_i, and **one or more pre-schooled aged children** (for whom $\text{dkidslt6}_i = 1$).

- First re-estimate Model 3 using the **dprobit** command with the **at(*vecname*)** option. The vector to use in the **at(*vecname*)** option is the vector x_{1i}^T containing the median values of the regressors in Model 3 when $\text{dkidslt6}_i = 1$:

$$x_{1i}^T = \left(\text{nwifeinc}_i \;\; \text{ed}_i \;\; \text{exp}_i \;\; \text{exp}_i^2 \;\; \text{age}_i \;\; 1 \;\; \text{nwifeinc}_i \;\; \text{ed}_i \;\; \text{exp}_i \;\; \text{exp}_i^2 \;\; \text{age}_i \;\; 1\right)$$

$$= \begin{pmatrix} \text{nwinc50p} & \text{ed50p} & \text{exp50p} & \text{expsq50p} & \text{age50p} & 1 \\ \text{nwinc50p} & \text{ed50p} & \text{exp50p} & \text{expsq50p} & \text{age50p} & 1 \end{pmatrix}$$

You previously created the vector x_{1i}^T and named it **x1median**. So simply enter the commands:

```
dprobit inlf nwifeinc ed exp expsq age dkidslt6 d6nwinc d6ed d6exp d6expsq d6age,
at(x1median)
ereturn list
display e(at)
```

Recall that the scalar **e(at)** contains the value of $\Phi\left(x_{1i}^T\hat{\beta}\right)$ generated by the previous **dprobit** command, where $\Phi\left(x_{1i}^T\hat{\beta}\right)$ is an estimate of $\Pr\left(\text{inlf}_i = 1 \mid \text{dkidslt6}_i = 1\right)$.

- Second, use the *Stata* statistical function **invnormal( )** to save the value of $x_{1i}^T\hat{\beta}$. Enter the commands:

```
scalar x1medbhat = invnormal(e(at))
scalar list x1medbhat
```

- Third, use the *Stata* statistical function **normalden( )** to save as a scalar the value of $\phi\left(x_{1i}^T\hat{\beta}\right)$, which is the standard normal density function (or p.d.f.) evaluated at $x_{1i}^T\hat{\beta}$. Enter the commands:

```
scalar phix1med = normalden(x1medbhat)
scalar list phix1med
```

These commands save the value of $\phi\left(x_{1i}^T\hat{\beta}\right)$ as the scalar **`phix1med`**.

- Compute the **estimated marginal *probability* effect of explanatory variable *nwifeinc$_i$*** for the ***median* married woman who has *one or more pre-school aged children***, which when **dkidslt6$_i$ = 1** is given by the function:

***estimated* marginal *probability* effect of nwifeinc$_i$** $= \phi\left(x_{1i}^T\hat{\beta}\right)\left(\hat{\beta}_1 + \hat{\beta}_7\right)$

Enter the **lincom** command:

```
lincom phix1med*(_b[nwifeinc] + _b[d6nwinc])
```

- Compute the **estimated marginal *probability* effect of explanatory variable *ed_i*** for the ***median* married woman who has *one or more pre-school aged children***, which when **$dkidslt6_i = 1$** is given by the function:

***estimated* marginal *probability* effect of ed_i** $= \phi\left(x_{1i}^T\hat{\beta}\right)\left(\hat{\beta}_2 + \hat{\beta}_8\right)$

Enter the **lincom** command:

```
lincom phix1med*(_b[ed] + _b[d6ed])
```

- Compute the **estimated marginal *probability* effect of explanatory variable *exp_i*** for the ***median* married woman who has *one or more pre-school aged children***, which when **$dkidslt6_i = 1$** is given by the function:

***estimated* marginal *probability* effect of exp_i** $= \phi\left(x_{1i}^T\hat{\beta}\right)\left(\hat{\beta}_3 + \hat{\beta}_9 + 2(\hat{\beta}_4 + \hat{\beta}_{10})\,\text{exp50p}\right)$

Enter the **lincom** command:

```
lincom phix1med*(_b[exp] + _b[d6exp] + 2*(_b[expsq] + _b[d6expsq])*exp50p)
```

- Compute the **estimated marginal *probability* effect of explanatory variable *age_i*** for the ***median* married woman who has *one or more pre-school aged children***, which when **$dkidslt6_i = 1$** is given by the function:

***estimated* marginal *probability* effect of age_i** $= \phi\left(x_i^T\beta\right)\left(\beta_5 + \beta_{11}\right)$

Enter the **lincom** command:

```
lincom phix1med*(_b[age] + _b[d6age])
```