ECON 452* -- NOTE 12

Testing for Heteroskedasticity in Linear Regression Models

This note identifies the ***two major forms of heteroskedasticity*** in linear regression models and explains commonly used procedures for testing for these two types of heteroskedasticity.

1. Forms of Heteroskedasticity

- The linear regression model is given by the population regression equation:

$$Y_i = \beta_0 + \sum_{j=1}^{k} \beta_j X_{ij} + u_i = x_i^T \beta + u_i \qquad (1.1)$$

$$y = X\beta + u \qquad (1.2)$$

where u_i is the i.d. (independently distributed) random error term that is suspected of being heteroskedastic.

- There are ***two*** **main forms of heteroskedasticity** in linear regression models for cross-section data:

(1) pure **heteroskedasticity**;

(2) mixed **heteroskedasticity**.

- **Pure heteroskedasticity** corresponds to error variances σ_i^2 of the form:

$$\sigma_i^2 = \sigma^2 Z_i^m \quad \text{where } Z_i^m > 0 \qquad \text{for all i} \qquad (2)$$

where:

- $\sigma^2 > 0$ is a finite (positive) constant;
- m is some known pre-specified number;
- Z_i is a known observable variable that may or may not be one of the regressors in the PRE under consideration.

- *Examples of Pure Heteroskedasticity*:

$$\sigma_i^2 = \sigma^2 Z_i, \qquad Z_i > 0 \qquad \text{for all i}$$

$$\sigma_i^2 = \sigma^2 Z_i^2 \qquad \text{for all i}$$

$$\sigma_i^2 = \sigma^2 Z_i^{-1} = \frac{\sigma^2}{Z_i}, \qquad Z_i > 0 \qquad \text{for all i}$$

❑ **Mixed heteroskedasticity** corresponds to error variances σ_i^2 of a very general form.

- ***Scalar formulation***

$$\sigma_i^2 = h(\alpha_0 + \alpha_1 Z_{i1} + \alpha_2 Z_{i2} + \cdots + \alpha_p Z_{ip}), \;\; h(\alpha_0) > 0 \qquad \text{for all i} \qquad (3)$$

where:

- $h(\cdot)$ denotes a **continuous positively-valued function** of some form, called the **conditional variance function**;
- the Z_{ij} (j = 1, ..., p) are known variables;
- the α_j (j = 0,1, ..., p) are unknown constant coefficients.

- ***Vector formulation***

$$\sigma_i^2 = h(z_i^T \alpha) > 0 \qquad \text{for all i} \qquad (3)$$

where:

- $h(\cdot)$ denotes a continuous positively-valued function of some form, called the **conditional variance function**;
- $z_i^T = (1 \quad Z_{i1} \quad Z_{i2} \quad \cdots \quad Z_{ip})$ is a 1×(p+1) row vector of known variables;
- $\alpha = (\alpha_0 \quad \alpha_1 \quad \alpha_2 \quad \cdots \quad \alpha_p)^T$ is a (p+1)×1 column vector of constant (unknown) coefficients.

- *Examples of Mixed Heteroskedasticity*:

$$\sigma_i^2 = \alpha_0 + \alpha_1 Z_{i1}, \quad \alpha_0 > 0 \qquad \text{for all i}$$

$$\sigma_i^2 = \left(\alpha_0 + \alpha_1 X_{ij} + \alpha_2 X_{ij}^2\right)^2 \qquad \text{for all i}$$

$$\sigma_i^2 = \exp\left(x_i^T \alpha\right) = \exp\left(\alpha_0 + \alpha_1 X_{i1} + \alpha_2 X_{i2} + \cdots + \alpha_k X_{ik}\right) \qquad \text{for all i}$$

2. LM Tests for Mixed Heteroskedasticity

❑ **Null and Alternative Hypotheses**

Consider the linear regression model for which the population regression equation can be written

(1) for the i-th sample observation as

$$Y_i = x_i^T \beta + u_i \tag{1.1}$$

(2) for all N sample observations as

$$y = X\beta + u \tag{1.2}$$

- **The *Null Hypothesis* of Homoskedastic Errors**

$$H_0\!: \ \mathrm{Var}\left(u_i \middle| x_i^T\right) = E\left(u_i^2 \middle| x_i^T\right) = \sigma^2 > 0 \qquad \forall\ i \tag{4.1}$$

where σ^2 is a finite positive (unknown) *constant*.

- **The *Alternative Hypothesis* of Mixed Heteroskedastic Errors**

$$H_1\!: \ \mathrm{Var}\left(u_i \middle| x_i^T\right) = E\left(u_i^2 \middle| x_i^T\right) = \sigma_i^2 = h\left(z_i^T \alpha\right) > 0 \qquad \forall\ i \tag{5.1}$$

where $h\left(z_i^T \alpha\right) = h\left(\alpha_0 + \alpha_1 Z_{i1} + \alpha_2 Z_{i2} + \cdots + \alpha_p Z_{ip}\right) > 0$ for all i.

- **Comparison of Null and Alternative Hypotheses**

$$H_0\text{: } Var(u_i|x_i^T) = E(u_i^2|x_i^T) = \sigma^2 > 0 \quad \forall\, i \tag{4.1}$$

$$H_1\text{: } Var(u_i|x_i^T) = E(u_i^2|x_i^T) = \sigma_i^2 = h(z_i^T\alpha) > 0 \quad \forall\, i \tag{5.1}$$

Comparing the null and alternative hypotheses H_0 and H_1 above, it is apparent that a test of the null hypothesis H_0 of homoskedastic errors against the alternative hypothesis H_1 of mixed heteroskedastic errors amounts to testing the null hypothesis

$$H_0\text{:} \quad \alpha_j = 0 \quad \forall\, j = 1, \ldots, p \tag{4.2}$$

against the alternative hypothesis

$$H_1\text{:} \quad \alpha_j \neq 0 \quad j = 1, \ldots, p \tag{5.2}$$

To see this, note that if all p coefficients α_j (j = 1, …, p) of the variables Z_{ij} (j = 1, …, p) equal zero, then the alternative hypothesis

$$\sigma_i^2 = h(z_i^T\alpha) = h(\alpha_0 + \alpha_1 Z_{i1} + \alpha_2 Z_{i2} + \cdots + \alpha_p Z_{ip}) > 0$$

reduces to

$$\sigma_i^2 = h(\alpha_0) > 0 \quad \text{where } h(\alpha_0) \text{ is a finite positive constant.}$$

In other words, if the p exclusion restrictions $\alpha_j = 0 \ \forall\, j = 1, \ldots, p$ are true, then the error variance is simply a finite positive constant, which means that the error terms u_i are homoskedastic.

❑ Rationale for Using an LM (Lagrange Multiplier) Test

- Recall that the **LM principle of hypothesis testing** performs an hypothesis test **using *only restricted* parameter estimates** of the model in question computed under the null hypothesis.

- An LM test for mixed heteroskedasticity would therefore compute the test statistic **using only OLS estimates of the model**.

  - This is a considerable practical convenience because estimating the model under the alternative hypothesis of mixed heteroskedasticity would require estimation procedures much more complicated than OLS.

  - Consequently, tests that require only the relatively simple and routine computations of OLS are substantially easier to perform than either Wald or Likelihood Ratio (LR) tests, both of which require estimation of the model under the alternative hypothesis of some specific form of mixed heteroskedasticity.

- The most widely used LM test for mixed heteroskedasticity is the **non-normality robust variant of the Breusch-Pagan test** proposed by Koenker.

  - The **original Breusch-Pagan LM test** for mixed heteroskedasticity depends crucially on **the assumption of error normality**. That is, the asymptotic null distribution of the original BP LM test statistic is chi-square with p degrees of freedom only if the random errors u_i are i.d. $N(0, \sigma_i^2)$ $\forall$ i = 1, …, N, where "i.d." means "independently distributed."

  - The advantage of the **Breusch-Pagan-Koenker LM test** is that the asymptotic null distribution of the BPK LM test statistic is $\chi^2[p]$ even if the error terms are not normally distributed. The BPK LM test requires only that the u_i are i.d. $(0, \sigma_i^2)$ $\forall$ i = 1, …, N, meaning that the random errors are independently distributed with zero mean and finite variances σ_i^2.

3. An LM Test for Mixed Heteroskedasticity: The BPK Test

❑ **The BPK LM Test Statistic for Mixed Heteroskedasticity**

We first present a general formula for Koenker's non-normality robust variant of the BP test statistic.

Since the test is based on the LM principle of hypothesis testing, it requires computation of restricted estimates of the model in question under the null hypothesis of homoskedastic errors.

These ***restricted* estimates are simply the *OLS estimates*** of the linear regression equation $Y_i = x_i^T\beta + u_i$, i = 1, …, N:

$\tilde{\beta} = (X^TX)^{-1}X^Ty$ = the **OLS estimator of coefficient vector** $\boldsymbol{\beta}$;

$\tilde{u} = y - X\tilde{\beta}$ = the **N×1 vector of OLS residuals**, the i-th element of which is $\tilde{u}_i = Y_i - x_i^T\tilde{\beta}$ (i = 1, …, N);

$\tilde{v}$ = the **N×1 vector of *squared* OLS residuals**, i-th element of which is $\tilde{v}_i = \tilde{u}_i^2 = (Y_i - x_i^T\tilde{\beta})^2$ (i = 1, …, N);

$\tilde{\sigma}^2_{ML} = \tilde{u}^T\tilde{u}/N = \sum_{i=1}^N \tilde{u}_i^2 / N$ = the **ML estimator of the constant error variance** $\boldsymbol{\sigma^2}$ under the null hypothesis.

Finally, let Z be the N×(p+1) matrix of observed sample values of the Z_{ij} variables that enter the conditional variance function under the alternative hypothesis of mixed heteroskedastic errors:

$$Z = \begin{bmatrix} z_1^T \\ z_2^T \\ z_3^T \\ \vdots \\ z_N^T \end{bmatrix} = \begin{bmatrix} 1 & Z_{11} & Z_{12} & \cdots & Z_{1k} \\ 1 & Z_{21} & Z_{22} & \cdots & Z_{2k} \\ 1 & Z_{31} & Z_{32} & \cdots & Z_{3k} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & Z_{N1} & Z_{N2} & \cdots & Z_{Nk} \end{bmatrix}.$$

- The **BPK LM test statistic, denoted as LM-BPK**, takes the form:

$$\text{LM–BPK} = N\,\frac{\tilde{v}^T Z\left(Z^T Z\right)^{-1} Z^T \tilde{v} \; - N\tilde{\sigma}^4_{ML}}{\tilde{v}^T \tilde{v} \; - N\tilde{\sigma}^4_{ML}} \qquad (6)$$

Remarks: The LM-BPK statistic (6) does not appear, at first glance, to be very easy to calculate. But there is in fact a simple way to do it.

- **<u>Computation of the LM-BPK Test Statistic</u>**

- The easiest way to compute the LM-BPK statistic (6) is to estimate by OLS an auxiliary regression equation called an **LM test regression**, and then calculate the sample value of LM-BPK from the results of this LM test regression.

- The **LM test regression for computing the LM-BPK test statistic** consists of an OLS regression of the *squared* OLS residuals from the original regression model, $\tilde{v}_i = \tilde{u}_i^2 = \left(Y_i - x_i^T\tilde{\beta}\right)^2$, i = 1, …, N, on all the Z_j variables that appear in the conditional variance function specified by the alternative hypothesis H_1.

  In vector-matrix form, the **LM test regression for the BPK test** takes the form

$$\tilde{v} = Zd + e \qquad (7.1)$$

where:

$\tilde{v}$ = the N×1 vector of *squared* OLS residuals, with i-th element $\tilde{v}_i = \tilde{u}_i^2 = \left(Y_i - x_i^T\tilde{\beta}\right)^2$, i = 1, …, N;

Z = the N×(p+1) matrix of observed sample values of the Z_j variables with i-th row $z_i^T = \left(1 \quad Z_{i1} \quad Z_{i2} \quad \cdots \quad Z_{ip}\right)$, i = 1, …, N;

d = $\left(d_0 \quad d_1 \quad d_2 \quad \cdots \quad d_p\right)^T$, a (p+1) ×1 column vector of coefficients;

e = an N×1 vector of errors e_i, i = 1, …, N.

In scalar form, the LM test regression for the BPK test can be written for the i-th sample observation as:

$$\tilde{v}_i = \tilde{u}_i^2 = d_0 + d_1 Z_{i1} + d_2 Z_{i2} + \cdots + d_p Z_{ip} + e_i = d_0 + \sum_{j=1}^{p} d_p Z_{ij} + e_i \qquad (7.2)$$

- **Computational formula for LM-BPK.** Given OLS estimates of the LM test regression (7.1)/(7.2), the LM-BPK test statistic can be computed as:

$$\text{LM–BPK} = N\,R^2_{\hat{u}^2} \qquad (8)$$

where:

$$R^2_{\hat{u}^2} = ESS_{\hat{u}^2} / TSS_{\hat{u}^2}$$
= the R^2 from OLS estimation of LM test regression (7);

$$ESS_{\hat{u}^2} = \tilde{v}^T Z \left(Z^T Z\right)^{-1} Z^T \tilde{v} - N\tilde{\sigma}^4_{ML}$$
= the explained sum-of-squares from OLS estimation of LM test regression (7);

$$TSS_{\hat{u}^2} = \tilde{v}^T \tilde{v} - N\tilde{\sigma}^4_{ML}$$
= the total sum-of-squares from OLS estimation of LM test regression (7).

❑ Asymptotic Null Distribution of the LM-BPK Test Statistic

- Recall the ***null*** **and** ***alternative*** **hypotheses**:

H_0: $\operatorname{Var}\left(u_i \middle| x_i^T\right) = E\left(u_i^2 \middle| x_i^T\right) = h(\alpha_0) = \sigma^2 > 0 \qquad \forall\ i$
implies $\alpha_j = 0 \quad \forall\ j = 1, \ldots, p$

H_1: $\operatorname{Var}\left(u_i \middle| x_i^T\right) = E\left(u_i^2 \middle| x_i^T\right) = \sigma_i^2 = h\left(z_i^T \alpha\right) > 0 \qquad \forall\ i$
implies $\alpha_j \neq 0 \quad j = 1, \ldots, p$

- The null hypothesis H_0 of homoskedastic errors **imposes p coefficient exclusion restrictions $\alpha_j = 0$** $\forall$ j = 1, …, p on the conditional variance function $h(z_i^T\alpha)$ = $h(\alpha_0 + \alpha_1 Z_{i1} + \alpha_2 Z_{i2} + \cdots + \alpha_p Z_{ip})$ specified by the alternative hypothesis H_1.

- The **asymptotic null distribution of the LM-BPK statistic is χ^2[p],** the chi-square distribution with p degrees of freedom:

$$\text{LM–BPK} \overset{a}{\sim} \chi^2[p] \quad \text{under } H_0. \tag{9}$$

where " $\overset{a}{\sim}$ " means "is asymptotically distributed as."

Remarks:

- Since the null distribution of the LM-BPK statistic is only asymptotically χ^2[p], **the BPK LM test is a *large sample* test**.
- The null distribution of LM-BPK is thus only *approximately* χ^2[p] in finite samples of any particular size N.

❑ An Alternative BPK Test Statistic

- In practice, an alternative test statistic -- the BPK F-statistic -- is often used in place of the LM-BPK test statistic in computing the BPK LM test for mixed heteroskedasticity.

- **The BPK F-statistic.** The **BPK F-statistic -- denoted as F-BPK --** is also calculated using the R-squared $R^2_{\hat{u}^2}$ from OLS estimation of the BPK LM test regression (7):

$$\text{F–BPK} = \frac{R^2_{\hat{u}^2}/p}{(1 - R^2_{\hat{u}^2})/(N - p - 1)} = \frac{R^2_{\hat{u}^2}}{(1 - R^2_{\hat{u}^2})}\frac{(N - p - 1)}{p} \tag{10}$$

Remarks:

- The F-BPK test statistic (10) is simply the conventional **analysis-of-variance F-statistic from OLS estimation of the LM test regression (7)**.

- This ANOVA-F statistic, which tests the joint significance of all p OLS slope coefficient estimates in the LM test regression, is routinely calculated by the OLS estimation commands in most econometric software programs.

- **Null distribution of F-BPK.** Under the null hypothesis of homoskedastic errors, the F-BPK statistic is distributed approximately as $F[p, N-p-1]$, the F-distribution with p numerator degrees of freedom and (N–p–1) denominator degrees of freedom:

$$\text{F–BPK} \overset{\text{app}}{\sim} F[p, N-p-1] \quad \text{under } H_0. \qquad (11)$$

where " $\overset{\text{app}}{\sim}$ " means "is approximately distributed as."

Summary of BPK LM Test Procedure for Mixed Heteroskedasticity

1. Estimate by OLS the original linear regression model

$$Y_i = x_i^T\beta + u_i \qquad \textit{or} \qquad y = X\beta + u$$

under the null hypothesis of homoskedastic errors to obtain the **OLS sample regression equation**

$$Y_i = x_i^T\tilde{\beta} + \tilde{u}_i \qquad \textit{or} \qquad y = X\tilde{\beta} + \tilde{u}$$

where

$\tilde{\beta} = \left(X^TX\right)^{-1}X^Ty$ = the **OLS estimator of coefficient vector** β;

$\tilde{u} = y - X\tilde{\beta}$ = the **N×1 OLS residual vector**, the i-th element of which is $\tilde{u}_i = Y_i - x_i^T\tilde{\beta}$, i = 1, …, N.

2. Save the OLS residuals $\tilde{u}_i = Y_i - x_i^T\tilde{\beta}$, i = 1, …, N, and **compute the *squared* OLS residuals** $\tilde{v}_i = \tilde{u}_i^2 = \left(Y_i - x_i^T\tilde{\beta}\right)^2$, i = 1, …, N.

3. Estimate by OLS the BPK LM test regression

$$\tilde{v} = Zd + e \qquad (7.1)$$

$$\tilde{v}_i = \tilde{u}_i^2 = d_0 + d_1 Z_{i1} + d_2 Z_{i2} + \cdots + d_p Z_{ip} + e_i = d_0 + \sum_{j=1}^{p} d_p Z_{ij} + e_i \qquad (7.2)$$

and **save the R-squared** from this test regression, $R^2_{\hat{u}^2} = ESS_{\hat{u}^2} / TSS_{\hat{u}^2}$.

4. Calculate the sample value of either the **LM-BPK test statistic (8)** or the **F-BPK test statistic (10)**, and **apply the conventional decision rule**.

<u>Test Statistics</u>:

$$\text{LM–BPK} = N R^2_{\hat{u}^2} \qquad (8)$$

$$\text{F–BPK} = \frac{R^2_{\hat{u}^2} / p}{(1 - R^2_{\hat{u}^2}) / (N - p - 1)} = \frac{R^2_{\hat{u}^2}}{(1 - R^2_{\hat{u}^2})} \frac{(N - p - 1)}{p} \qquad (10)$$

<u>Decision Rule</u>: Let $\chi^2_\alpha[p]$ denote the α-level critical value of the $\chi^2[p]$ distribution, and $F_\alpha[p, N-p-1]$ the α-level critical value of the $F[p, N-p-1]$ distribution.

Reject **the null hypothesis H_0** of homoskedastic errors at significance level α if

- p-value of LM-BPK < α *or* sample value of LM-BPK > $\chi^2_\alpha[p]$
- p-value of F-BPK < α *or* sample value of F-BPK > $F_\alpha[p, N-p-1]$

Retain **the null hypothesis H_0** of homoskedastic errors at significance level α if

- p-value of LM-BPK ≥ α *or* sample value of LM-BPK ≤ $\chi^2_\alpha[p]$
- p-value of F-BPK ≥ α *or* sample value of F-BPK ≤ $F_\alpha[p, N-p-1]$

4. A Test for Pure Heteroskedasticity: The Goldfeld-Quandt Test

- Suppose you wish to test for the presence of ***pure*** **heteroskedasticity** of the form $\sigma_i^2 = \sigma^2 Z_i^m$ (i = 1, ..., N) in the linear regression model

$$Y_i = \beta_0 + \sum_{j=1}^{k} \beta_j X_{ij} + u_i = x_i^T \beta + u_i \qquad (1.1)$$

$$y = X\beta + u \qquad (1.2)$$

where u_i is the i.d. (independently distributed) random error term that is suspected of being heteroskedastic.

❑ **Null and Alternative Hypotheses**

- **The *Null Hypothesis* of Homoskedastic Errors**

$$H_0\text{: } \sigma_i^2 = \sigma^2 > 0 \qquad \forall\ i \qquad (12)$$

where σ^2 is a finite positive (unknown) *constant*.

- **The *Alternative Hypothesis* of Pure Heteroskedastic Errors**

$$H_1\text{: } \sigma_i^2 = \sigma^2 Z_i^m, \quad \sigma^2 > 0, \quad Z_i^m > 0 \qquad \forall\ i \qquad (13)$$

❑ **The Goldfeld-Quandt (G-Q) Test Procedure**

The **Goldfeld-Quandt (G-Q) test** for this form of pure heteroskedasticity consists of the following five steps:

- **Step 1: Sort the sample observations in *ascending order*** according to the values of Z_i^m, from lowest to highest.

- **<u>Step 2</u>: Omit from the sorted sample *c* central observations**, where *c* is arbitrarily chosen to equal some value between N/6 and N/3. This defines two subsamples of the original sample: (1) a subsample of low-Z_i^m observations, containing N_L observations; and (2) a subsample of high-Z_i^m observations, containing N_H observations. Usually, the value of *c* is chosen so that each of the two subsamples contains (N – c)/2 observations, so that $N_L = N_H = (N - c)/2$.

- **<u>Step 3</u>: Estimate separately by OLS the regression equation (1) on each of the two subsamples**, and retrieve from each subsample regression the corresponding sum of squared OLS residuals. Let

  RSS_L = RSS from the OLS regression on the low-Z_i^m subsample;
  RSS_H = RSS from the OLS regression on the high-Z_i^m subsample.

  <u>Note</u>: Both RSS_L and RSS_H have the same degrees of freedom when the two subsamples have the same number of observations. That is, in the special case when $N_L = N_H = (N - c)/2$, $df_L = df_H = (N - c)/2 - K = (N - c - 2K)/2$.

- **<u>Step 4</u>: Compute the *sample value* of the G-Q test statistic**

$$F_{GQ} = \frac{\hat{\sigma}_H^2}{\hat{\sigma}_L^2} = \frac{RSS_H / df_H}{RSS_L / df_L} = \frac{RSS_H / (N_H - K)}{RSS_L / (N_L - K)}. \tag{14}$$

♦ ***Special Case of G-Q Test Statistic***

In the special case when $N_H = N_L = (N - c)/2$, i.e., when the two subsamples have the same number of observations, both RSS_H and RSS_L have degrees of freedom equal to (N – c – 2K)/2; in this case, the G-Q test statistic in (14) takes the form

$$F_{GQ} = \frac{\hat{\sigma}_H^2}{\hat{\sigma}_L^2} = \frac{RSS_H / (N_H - K)}{RSS_L / (N_L - K)} = \frac{RSS_H}{RSS_L}, \tag{15}$$

since $(N_H - K) = (N_L - K) = (N - c - 2K)/2$ and the (N – c – 2K)/2 term cancels out of the numerator and denominator of F_{GQ}.

- ***Null Distribution*** **of the G-Q Test Statistic F_{GQ}**

  Under the null hypothesis H_0 and the assumption that the errors u_i are normally distributed, the F_{GQ} statistic in (14) and (15) is distributed as the F-distribution with numerator degrees of freedom = $N_H - K$ and denominator degrees of freedom = $(N_L - K)$: i.e., **under H_0, the null distribution of F_{GQ} is**

$$F_{GQ} \sim F\big[(N_H - K), (N_L - K)\big] = F\big[(N - c - 2K)/2, (N - c - 2K)/2\big] \qquad (16)$$

  where the second equality is appropriate in the special case when $(N_H - K) = (N_L - K) = (N - c - 2K)/2$.

- **Step 5: Apply the conventional decision rule for an F-test**.

- **Decision Rule:** Let $F_\alpha\big[(N_H - K), (N_L - K)\big]$ denote the α-level (or 100α percent) critical value of the $F\big[(N_H - K), (N_L - K)\big]$-distribution.

  Reject **the null hypothesis H_0** of homoskedastic errors at significance level α if

  p-value of $F_{GQ} < \alpha$ *or* sample value of $F_{GQ} > F_\alpha\big[(N_H - K), (N_L - K)\big]$

  Retain **the null hypothesis H_0** of homoskedastic errors at significance level α if

  p-value of $F_{GQ} \geq \alpha$ *or* sample value of $F_{GQ} \leq F_\alpha\big[(N_H - K), (N_L - K)\big]$

- **Interpretation of the Decision Rule:** Note that if the alternative hypothesis H_1 is true, the calculated value of F_{GQ} will tend to be large. The reason for this is that, according to the alternative hypothesis H_1, the values of σ_i^2 are larger for the high-Z_i^m subsample than for the low-Z_i^m subsample. Hence the residual sum of squares RSS_H for the high-Z_i^m subsample will tend to be large relative to the residual sum of squares RSS_L for the low-Z_i^m subsample if the alternative hypothesis is in fact true.