ECON 452* -- NOTE 8

A General Regression Model with Dummy Variable Interactions

Starting Point: Model 5.4 in Standard Notation

- The **population regression equation for Model 5.4** can be written in standard notation as:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 IN2_i + \beta_4 IN3_i + \beta_5 IN4_i + \delta_0 F_i$$
$$+ \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i IN2_i + \delta_4 F_i IN3_i + \delta_5 F_i IN4_i + u_i \qquad \textbf{(5.4)}$$

- The **population regression *function* for Model 5.4** is:

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$
$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 IN2_i + \beta_4 IN3_i + \beta_5 IN4_i + \delta_0 F_i$$
$$+ \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i IN2_i + \delta_4 F_i IN3_i + \delta_5 F_i IN4_i \qquad (5.4')$$

- The ***female* population regression function for Model 5.4** is obtained by setting the female indicator $F_i = 1$ in (5.4'):

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$
$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 IN2_i + \beta_4 IN3_i + \beta_5 IN4_i + \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 IN2_i + \delta_4 IN3_i + \delta_5 IN4_i$$
$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1) X_{i1} + (\beta_2 + \delta_2) X_{i2} + (\beta_3 + \delta_3) IN2_i + (\beta_4 + \delta_4) IN3_i + (\beta_5 + \delta_5) IN4_i \qquad \textbf{(5.4f)}$$

$$E(Y_i \mid X_{i1}, X_{i2}, X_{i3}, F_i, IN2_i, IN3_i, IN4_i)$$
$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 IN2_i + \beta_5 IN3_i + \beta_6 IN4_i + \delta_0 F_i$$
$$+ \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i3} + \delta_4 F_i IN2_i + \delta_5 F_i IN3_i + \delta_6 F_i IN4_i \qquad (5.4')$$

- The ***male*** **population regression function for Model 5.4** is obtained by setting the female indicator $F_i = 0$ in (5.4'):

$$E(Y_i \mid F_i = 0, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i) = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 IN2_i + \beta_4 IN3_i + \beta_5 IN4_i \qquad \textbf{(5.4m)}$$

- The ***female-male difference*** **in conditional mean Y for Model 5.4** is obtained by subtracting the male regression function (5.4m) from the female regression function (5.4f):

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$
$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 IN2_i + \beta_4 IN3_i + \beta_5 IN4_i + \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 IN2_i + \delta_4 IN3_i + \delta_5 IN4_i$$
$$- \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2} - \beta_3 IN2_i - \beta_4 IN3_i - \beta_5 IN4_i$$
$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 IN2_i + \delta_4 IN3_i + \delta_5 IN4_i$$

<u>Model 5.5</u>: Quadratic terms in the continuous explanatory variables X_1 and X_2

Expand Model 5.4 to include quadratic terms in the two continuous explanatory variables X_1 and X_2.

The **population regression *equation* for Model 5.5** is:

$$\begin{aligned} Y_i = {} & \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i \\ & + \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i + u_i \qquad \textbf{(5.5)} \end{aligned}$$

Estimate by OLS the population regression *equation* for Model 5.5 using the following ***Stata*** command:

```
regress y x1 x2 x1sq x2sq x1x2 in2 in3 in4 f fx1 fx2 fx1sq fx2sq fx1x2 fin2
fin3 fin4
```

The **population regression *function* for Model 5.5** is:

$$\begin{aligned} & E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i) \\ & = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i \\ & \quad + \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \qquad \textbf{(5.5')} \end{aligned}$$

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \quad \textbf{(5.5')}$$

- The ***female*** **population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 1$ in (5.5'):

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1) X_{i1} + (\beta_2 + \delta_2) X_{i2} + (\beta_3 + \delta_3) X_{i1}^2 + (\beta_4 + \delta_4) X_{i2}^2 + (\beta_5 + \delta_5) X_{i1} X_{i2}$$

$$+ (\beta_6 + \delta_6) IN2_i + (\beta_7 + \delta_7) IN3_i + (\beta_8 + \delta_8) IN4_i \quad \textbf{(5.5f)}$$

- The ***male*** **population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 0$ in (5.5'):

$$E(Y_i \mid F_i = 0, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

- The ***female-male difference*** **in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) \; - \; E(Y_i \mid F_i = 0, x_i^T)$$

$$\begin{aligned}
= \; & \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i \\
& + \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i \\
& - \left(\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i \right)
\end{aligned}$$

$$\begin{aligned}
= \; & \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i \\
& + \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i \\
& - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2} - \beta_3 X_{i1}^2 - \beta_4 X_{i2}^2 - \beta_5 X_{i1} X_{i2} - \beta_6 IN2_i - \beta_7 IN3_i - \beta_8 IN4_i
\end{aligned}$$

$$= \; \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

- **Table formats** for reporting coefficient estimates of **Model 5.5**:

Table 2: OLS Estimates of Model 5.5 on Pooled Sample of Females and Males

	Females		Males		Female-Male Differences	
Regressor Name	Coef. Estimate	t-ratio	Coef. Estimate	t-ratio	Coef. Estimate	t-ratio
Intercept						
X_1						
X_2						
X_1-sq						
X_2-sq						
X_1X_2						
IN2						
IN3						
IN4						
No. of obs =						
RSS =						
R-squared =						
ANOVA F =						
p-value of F =						

Table 2: OLS Estimates of Model 5.5 on Pooled Sample of Females and Males

	Females		Males		Female-Male Differences	
Regressor	$\hat{\beta}_j + \hat{\delta}_j$	t-ratio	$\hat{\beta}_j$	t-ratio	$\hat{\delta}_j$	t-ratio
Intercept						
X_1						
X_2						
X_1-sq						
X_2-sq						
X_1X_2						
IN2						
IN3						
IN4						
No. of obs =						
RSS =						
R-squared =						
ANOVA F =						
p-value of F =						

Table 5: Hypothesis Test Results for Model 5.5

#	Null Hypothesis H_0	Interpretation of H_0	q[1/]	p-value[2/]
1	$\beta_1 + \delta_1 = 0$ & $\beta_3 + \delta_3 = 0$ & $\beta_5 + \delta_5 = 0$	ME of X_1 is zero for females	3	0.0000
2	$\beta_3 + \delta_3 = 0$ & $\beta_5 + \delta_5 = 0$	ME of X_1 is constant for females	2	0.0274
3	$\beta_1 = 0$ & $\beta_3 = 0$ & $\beta_5 = 0$	ME of X_1 is zero for males	3	0.0014
4	$\beta_3 = 0$ & $\beta_5 = 0$	ME of X_1 is constant for males	2	0.0083
5	$\delta_1 = 0$ & $\delta_3 = 0$ & $\delta_5 = 0$	ME of X_1 equal for females & males	3	0.0038
6	$\delta_3 = 0$ & $\delta_5 = 0$	F-M difference in ME of X_1 a constant	2	0.1494
7	$\beta_2 + \delta_2 = 0$ & $\beta_4 + \delta_4$ & $\beta_5 + \delta_5 = 0$	ME of X_2 is zero for females	3	0.0000
8	$\beta_4 + \delta_4$ & $\beta_5 + \delta_5 = 0$	ME of X_2 is constant for females	2	0.0000
9	$\beta_2 = 0$ & $\beta_4 = 0$ & $\beta_5 = 0$	ME of X_2 is zero for males	3	0.0741
10	$\beta_4 = 0$ & $\beta_5 = 0$	ME of X_2 is constant for males	2	0.3185
11	$\delta_2 = 0$ & $\delta_4 = 0$ & $\delta_5 = 0$	ME of X_2 equal for females & males	3	0.0063
12	$\delta_4 = 0$ & $\delta_5 = 0$	F-M difference in ME of X_2 a constant	2	0.03119
13	$\beta_6 = 0$ & $\beta_7 = 0$ & $\beta_8 = 0$	No industry effects for males	3	0.0003
14	$\beta_6 + \delta_6 = 0$ & $\beta_7 + \delta_7$ & $\beta_8 + \delta_8 = 0$	No industry effects for females	3	0.0000
15	$\delta_6 = 0$ & $\delta_7 = 0$ & $\delta_8 = 0$	Industry effects equal, females & males	3	0.0083
16	$\delta_j = 0$ for all j = 0, 1, …, 8	F-M mean Y difference = 0	10	0.0000
17	$\delta_j = 0$ for all j = 1, …, 8	F-M mean Y difference is constant	9	0.0000

Notes: 1/. q denotes the number of coefficient restrictions specified by the null hypothesis H_0. 2/. The p-values are two-tail p-values for the calculated sample value of the test statistic.

The Marginal Effect of X_1 in Model 5.5

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \qquad \textbf{(5.5')}$$

- The **marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial Y_i}{\partial X_{i1}} = \frac{\partial E\left(Y_i \mid F_i, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 F_i + 2\delta_3 F_i X_{i1} + \delta_5 F_i X_{i2}$$

- The **marginal effect of X_1 for *females* in Model 5.5** is obtained by setting $F_i = 1$:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=1} = \frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

$$= (\beta_1 + \delta_1) + 2(\beta_3 + \delta_3) X_{i1} + (\beta_5 + \delta_5) X_{i2}$$

- The **marginal effect of X_1 for *males* in Model 5.5** is obtained by setting $F_i = 0$:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=0} = \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2}$$

- The ***female-male difference*** **in the marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial E\left(Y_i \,\middle|\, F_i = 1, x_i^T\right)}{\partial X_{i1}} - \frac{\partial E\left(Y_i \,\middle|\, F_i = 0, x_i^T\right)}{\partial X_{i1}}$$

$$= \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2} - \beta_1 - 2\beta_3 X_{i1} - \beta_5 X_{i2}$$

$$= \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

Hypothesis Tests Respecting the Marginal Effect of X_1 for *Males* in Model 5.5

- The **marginal effect of X_1 for *males* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=0} = \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2}$$

- ***Test 1m:*** Test the hypothesis that the ***marginal* effect of X_1 on Y for *males* is *zero*** for all values of X_1 and X_2.

- Sufficient conditions for $\partial Y_i / \partial X_{i1} = 0$ for all i for males are $\beta_1 = 0$ *and* $\beta_3 = 0$ *and* $\beta_5 = 0$.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\beta_1 = 0$ *and* $\beta_3 = 0$ *and* $\beta_5 = 0$

  H_1: $\beta_1 \neq 0$ *and/or* $\beta_3 \neq 0$ *and/or* $\beta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test x1 x1sq x1x2
```

- The **marginal effect of X_1 for *males* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=0} = \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2}$$

♦ ***Test 2m:*** Test the hypothesis that the ***marginal* effect of X_1 on Y for *males* is *constant*** – i.e., is unrelated to the values of X_1 and X_2.

- Sufficient conditions for $\partial Y_i / \partial X_{i1} = \beta_1$ (a constant) for all males are $\beta_3 = 0$ *and* $\beta_5 = 0$.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\beta_3 = 0$ *and* $\beta_5 = 0$

  H_1: $\beta_3 \neq 0$ *and/or* $\beta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test x1sq x1x2
```

- ***<u>Test 3m</u>:*** Test the hypothesis that the **marginal effect of X_1 on Y for *males* is unrelated to**, or **does not depend upon**, X_1.

- The **marginal effect of X_1 for *males* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=0} = \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2}$$

- A sufficient condition for $\partial Y_i / \partial X_{i1}$ to be unrelated to X_{i1} for all males is $\beta_3 = 0$.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\beta_3 = 0$

  H_1: $\beta_3 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

  `test x1sq` *or* **`test x1sq = 0`**

- Equivalently, compute a **two-tail t-test** of H_0 against H_1 using the following *Stata* **lincom** command:

```
lincom _b[x1sq]
```

- ***Test 4m:*** Test the hypothesis that the **marginal effect of X_1 on Y for *males* is unrelated to**, or **does not depend upon**, X_2.

- The **marginal effect of X_1 for *males* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=0} = \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2}$$

- A sufficient condition for $\partial Y_i / \partial X_{i1}$ to be unrelated to X_{i2} for all males is $\beta_5 = 0$.

- The ***null* and *alternative* hypotheses** for this proposition are:

  H_0: $\beta_5 = 0$

  H_1: $\beta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

  `test x1x2` *or* `test x1x2 = 0`

- Equivalently, compute a **two-tail t-test** of H_0 against H_1 using the following *Stata* **lincom** command:

```
lincom _b[x1x2]
```

Hypothesis Tests Respecting the Marginal Effect of X_1 for *Females* in Model 5.5

- The **marginal effect of X_1 for *females* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=1} = \frac{\partial E\left(Y_i \,\middle|\, F_i = 1, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

$$= (\beta_1 + \delta_1) + 2(\beta_3 + \delta_3)X_{i1} + (\beta_5 + \delta_5)X_{i2}$$

- ***Test 1f:*** Test the hypothesis that the ***marginal* effect of X_1 on Y for *females* is *zero*** for all values of X_1 and X_2.

- Sufficient conditions for $\partial Y_i / \partial X_{i1} = 0$ for all females are $\beta_1 + \delta_1 = 0$ *and* $\beta_3 + \delta_3 = 0$ *and* $\beta_5 + \delta_5 = 0$.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\beta_1 + \delta_1 = 0$ *and* $\beta_3 + \delta_3 = 0$ *and* $\beta_5 + \delta_5 = 0$

  H_1: $\beta_1 + \delta_1 \neq 0$ *and/or* $\beta_3 + \delta_3$ *and/or* $\beta_5 + \delta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** commands:

```
test x1 + fx1 = 0, notest
test x1sq + fx1sq = 0, accumulate notest
test x1x2 + fx1x2 = 0, accumulate
```

- ***Test 2f:*** Test the hypothesis that the ***marginal* effect of X_1 on Y for *females* is *constant*** – i.e., is unrelated to the values of X_1 and X_2.

- The **marginal effect of X_1 for *females* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=1} = \frac{\partial E\left(Y_i \,\middle|\, F_i = 1, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

$$= (\beta_1 + \delta_1) + 2(\beta_3 + \delta_3)X_{i1} + (\beta_5 + \delta_5)X_{i2}$$

- Sufficient conditions for $\partial Y_i / \partial X_{i1} = \beta_1 + \delta_1$ (a constant) for all females are $\beta_3 + \delta_3 = 0$ *and* $\beta_5 + \delta_5 = 0$.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\beta_3 + \delta_3 = 0$ *and* $\beta_5 + \delta_5 = 0$

  H_1: $\beta_3 + \delta_3 \neq 0$ *and/or* $\beta_5 + \delta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test x1sq + fx1sq = 0, notest
test x1x2 + fx1x2 = 0, accumulate
```

- ***Test 3f:*** Test the hypothesis that the **marginal effect of X_1 on Y for *females* is unrelated to**, or **does not depend upon**, X_1.

- The **marginal effect of X_1 for *females* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=1} = \frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

$$= (\beta_1 + \delta_1) + 2(\beta_3 + \delta_3)X_{i1} + (\beta_5 + \delta_5)X_{i2}$$

- A sufficient condition for $\partial Y_i / \partial X_{i1}$ to be unrelated to X_{i1} for all females is $\beta_3 + \delta_3 = 0$.

- The ***null*** **and *alternative* hypotheses** are:

H_0: $\beta_3 + \delta_3 = 0$

H_1: $\beta_3 + \delta_3 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test x1sq + fx1sq = 0
```

- Equivalently, compute a **two-tail t-test** of H_0 against H_1 using the following *Stata* **lincom** command:

```
lincom _b[x1sq] + _b[fx1sq]
```

- ***Test 4f:*** Test the hypothesis that the **marginal effect of X_1 on Y for *females* is unrelated to**, or **does not depend upon**, **X_2**.

- The **marginal effect of X_1 for *females* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=1} = \frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

$$= (\beta_1 + \delta_1) + 2(\beta_3 + \delta_3)X_{i1} + (\beta_5 + \delta_5)X_{i2}$$

- A sufficient condition for $\partial Y_i / \partial X_{i1}$ to be unrelated to X_{i2} for all females is $\beta_5 + \delta_5 = 0$.

- The ***null* and *alternative* hypotheses** for this proposition are:

  H_0: $\beta_5 + \delta_5$

  H_1: $\beta_5 + \delta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test x1x2 + fx1x2 = 0
```

- Equivalently, compute a **two-tail t-test** of H_0 against H_1 using the following *Stata* **lincom** command:

```
lincom _b[x1x2] + _b[fx1x2]
```

Hypothesis Tests for *Female-Male Differences* in the Marginal Effect of X_1 in Model 5.5

- The ***female-male difference* in the marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} - \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

♦ ***<u>Test 5</u>:*** Test the hypothesis that the ***marginal* effect of X_1 on Y for *females* equals** the ***marginal* effect of X_1 on Y for *males*** for any values of X_1 and X_2 **–** i.e., the ***female-male difference*** in the ***marginal* effect of X_1 on Y is *zero*** for any values of X_1 and X_2.

- Sufficient conditions for the **female-male *difference*** in the marginal effect of X_1 on Y **to equal *zero*** for all values of X_1 and X_2 are $\delta_1 = 0$ *and* $\delta_3 = 0$ *and* $\delta_5 = 0$.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\delta_1 = 0$ *and* $\delta_3 = 0$ *and* $\delta_5 = 0$

  H_1: $\delta_1 \neq 0$ *and/or* $\delta_3 \neq 0$ *and/or* $\delta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fx1 fx1sq fx1x2
```

- **Remark:** The null hypothesis H_0 implies that the female-male difference in conditional mean Y is unrelated to the explanatory variable X_1.

  The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

$$= \delta_0 + \delta_2 X_{i2} + \delta_4 X_{i2}^2 + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$ under H_0: $\delta_1 = 0$ *and* $\delta_3 = 0$ *and* $\delta_5 = 0$

- ***Test 6:*** Test the hypothesis that the ***female-male difference*** in the ***marginal* effect of X_1 on Y is *a constant*** for any values of X_1 and X_2.

- The ***female-male difference* in the marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} - \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

- Sufficient conditions for the **female-male *difference*** in the marginal effect of X_1 on Y **to equal the constant δ_1** for all values of X_1 and X_2 are $\delta_3 = 0$ *and* $\delta_5 = 0$.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\delta_3 = 0$ *and* $\delta_5 = 0$

  H_1: $\delta_3 \neq 0$ *and/or* $\delta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fx1sq fx1x2
```

- **Remark:** The null hypothesis H_0 implies that the female-male difference in conditional mean Y is unrelated to the regressors X_{i1}^2 and $X_{i1}X_{i2}$.

  The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_4 X_{i2}^2 + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i \qquad \text{under } H_0\text{: } \delta_3 = 0 \textit{ and } \delta_5 = 0$$

- ***Test 7:*** Test the hypothesis that the ***female-male difference*** in the ***marginal*** **effect of X_1 on Y is unrelated to**, or **does not depend upon**, $\mathbf{X_1}$.

- The ***female-male difference*** **in the marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial E\left(Y_i \,\middle|\, F_i = 1, x_i^T\right)}{\partial X_{i1}} - \frac{\partial E\left(Y_i \,\middle|\, F_i = 0, x_i^T\right)}{\partial X_{i1}} = \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

- A sufficient condition for the **female-male *difference*** in the marginal effect of X_1 on Y **to be unrelated to X_1** is $\delta_3 = 0$.

- The ***null*** **and *alternative* hypotheses** are:

  H_0: $\delta_3 = 0$

  H_1: $\delta_3 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fx1sq
```
or
```
test fx1sq = 0
```

- Equivalently, compute a **two-tail t-test** of H_0 against H_1 using the following *Stata* **lincom** command:

```
lincom _b[fx1sq]
```

- **Remark:** The null hypothesis H_0 implies that the female-male difference in conditional mean Y is unrelated to the regressor X_{i1}^2.

  The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i \qquad \text{under } H_0\text{: } \delta_3 = 0$$

- ***Test 8:*** Test the hypothesis that the ***female-male difference*** in the ***marginal*** **effect of X_1 on Y is unrelated to**, or **does not depend upon**, X_2.

- The ***female-male difference*** **in the marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial E\left(Y_i \,\middle|\, F_i = 1, x_i^T\right)}{\partial X_{i1}} - \frac{\partial E\left(Y_i \,\middle|\, F_i = 0, x_i^T\right)}{\partial X_{i1}} = \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

- A sufficient condition for the **female-male *difference*** in the marginal effect of X_1 on Y **to be unrelated to X_2** is $\delta_5 = 0$.

- The ***null*** **and *alternative* hypotheses** are:

H_0: $\delta_5 = 0$

H_1: $\delta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fx1x2          or          test fx1x2 = 0
```

- Equivalently, compute a **two-tail t-test** of H_0 against H_1 using the following *Stata* **lincom** command:

```
lincom _b[fx1x2]
```

- **Remark:** The null hypothesis H_0 implies that the female-male difference in conditional mean Y is unrelated to the regressor $X_{i1}X_{i2}$.

  The ***female-male difference* in conditional mean Y for Model 5.5** is:

  $$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

  $$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

  $$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i \qquad \text{under } H_0\text{: } \delta_5 = 0$$

Hypothesis Tests Respecting the Effects of Industry in Model 5.5

- ***Test 1-Industry:*** Test the hypothesis of **no industry effects for *males***. This is equivalent to the hypothesis that conditional mean Y for males is unrelated to industry, i.e., that there **are no inter-industry differences in conditional mean Y for *males***.

- The ***male*** **population regression function for Model 5.5** is:

$$E(Y_i \mid F_i = 0, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

- Sufficient conditions for the conditional mean value of Y for males to be unrelated to industry are $\beta_6 = 0$ *and* $\beta_7 = 0$ *and* $\beta_8 = 0$.

- The ***null*** **and *alternative* hypotheses** are:

H_0: $\beta_6 = 0$ *and* $\beta_7 = 0$ *and* $\beta_8 = 0$

H_1: $\beta_6 \neq 0$ *and/or* $\beta_7 \neq 0$ *and/or* $\beta_8 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test in2 in3 in4
```

- ***Test 2-Industry:*** Test the hypothesis of **no industry effects for *females***. This is equivalent to the hypothesis that conditional mean Y for females is unrelated to industry, i.e., that there **are no inter-industry differences in conditional mean Y for *females***.

- The ***female* population regression function for Model 5.5** is:

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$
$$+ \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1) X_{i1} + (\beta_2 + \delta_2) X_{i2} + (\beta_3 + \delta_3) X_{i1}^2 + (\beta_4 + \delta_4) X_{i2}^2 + (\beta_5 + \delta_5) X_{i1} X_{i2}$$
$$+ (\beta_6 + \delta_6) IN2_i + (\beta_7 + \delta_7) IN3_i + (\beta_8 + \delta_8) IN4_i \qquad \textbf{(5.5f)}$$

- Sufficient conditions for the conditional mean value of Y for females to be unrelated to industry are $\beta_6 + \delta_6 = 0$ *and* $\beta_7 + \delta_7 = 0$ *and* $\beta_8 + \delta_8 = 0$.

- The ***null* and *alternative* hypotheses** are:

H_0: $\beta_6 + \delta_6 = 0$ *and* $\beta_7 + \delta_7 = 0$ *and* $\beta_8 + \delta_8 = 0$

H_1: $\beta_6 + \delta_6 \neq 0$ *and/or* $\beta_7 + \delta_7 \neq 0$ *and/or* $\beta_8 + \delta_8 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** commands:

```
test in2 + fin2 = 0, notest
test in3 + fin3 = 0, accumulate notest
test in4 + fin4 = 0, accumulate
```

- ***Test 3-Industry:*** Test the hypothesis of **no *female-male differences* in industry effects** – i.e., that the ***female-male difference*** in conditional mean Y **is unrelated to industry**.

  This is equivalent to the hypothesis that **industry effects are *equal*** for ***females* and *males***, i.e., that **inter-industry differences in conditional mean Y for *females* equal inter-industry differences in conditional mean Y for *males***.

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

- Sufficient conditions for the female-male difference in conditional mean Y to be unrelated to industry (for equal industry effects for males and females) are $\delta_6 = 0$ *and* $\delta_7 = 0$ *and* $\delta_8 = 0$.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\delta_6 = 0$ *and* $\delta_7 = 0$ *and* $\delta_8 = 0$

  H_1: $\delta_6 \neq 0$ *and/or* $\delta_7 \neq 0$ *and/or* $\delta_8 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fin2 fin3 fin4
```

Hypothesis Tests Respecting *Female-Male Differences* in Conditional Mean Y in Model 5.5

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

♦ ***<u>Test 1</u>:*** The ***female-male difference* in conditional mean Y *equals zero* for all observations**, i.e., for any given values of the explanatory variables X_1, X_2, X_3, and industry.

- The ***null* and *alternative* hypotheses** are:

H_0: $\delta_0 = 0$ *and* $\delta_1 = 0$ *and* $\delta_2 = 0$ *and* $\delta_3 = 0$ *and* $\delta_4 = 0$ *and* $\delta_5 = 0$ *and* $\delta_6 = 0$ *and* $\delta_7 = 0$ *and* $\delta_8 = 0$

or

$\delta_j = 0$ for all j = 0, 1, …, 8

H_1: $\delta_0 \neq 0$ *and/or* $\delta_1 \neq 0$ *and/or* $\delta_2 \neq 0$ *and/or* $\delta_3 \neq 0$ *and/or* $\delta_4 \neq 0$ *and/or* $\delta_5 \neq 0$ *and/or* $\delta_6 \neq 0$ *and/or* $\delta_7 \neq 0$ *and/or* $\delta_8 \neq 0$

or

$\delta_j \neq 0$ j = 0, 1, …, 8

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test f fx1 fx2 fx1sq fx2sq fx1x2 fin2 fin3 fin4
```

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

♦ ***Test 2:*** The ***female-male difference* in conditional mean Y *equals a constant***, i.e., it **does not depend on** the values of the explanatory variables **X_1, X_2, X_3, and industry**.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\delta_1 = 0$ *and* $\delta_2 = 0$ *and* $\delta_3 = 0$ *and* $\delta_4 = 0$ *and* $\delta_5 = 0$ *and* $\delta_6 = 0$ *and* $\delta_7 = 0$ *and* $\delta_8 = 0$

  or

  $\delta_j = 0$ for all j = 1, 2, …, 8

  H_1: $\delta_1 \neq 0$ *and/or* $\delta_2 \neq 0$ *and/or* $\delta_3 \neq 0$ *and/or* $\delta_4 \neq 0$ *and/or* $\delta_5 \neq 0$ *and/or* $\delta_6 \neq 0$ *and/or* $\delta_7 \neq 0$ *and/or* $\delta_8 \neq 0$

  or

  $\delta_j \neq 0$ j = 1, 2, …, 8

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fx1 fx2 fx1sq fx2sq fx1x2 fin2 fin3 fin4
```

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

♦ ***Test 3:*** The ***female-male difference*** in conditional mean Y **does not depend on X_1** – i.e., the **marginal effect of X_1 is *equal*** for ***males* and *females***.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\delta_1 = 0$ *and* $\delta_3 = 0$ *and* $\delta_5 = 0$

  H_1: $\delta_1 \neq 0$ *and/or* $\delta_3 \neq 0$ *and/or* $\delta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fx1 fx1sq fx1x2
```

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

♦ ***Test 4:*** The ***female-male difference*** in conditional mean Y **does not depend on X_2** – i.e., the **marginal effect of X_2 is *equal*** for ***males* and *females***.

- The ***null* and *alternative* hypotheses** are:

  H_0: $\delta_2 = 0$ *and* $\delta_4 = 0$ *and* $\delta_5 = 0$

  H_1: $\delta_2 \neq 0$ *and/or* $\delta_4 \neq 0$ *and/or* $\delta_5 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fx2 fx2sq fx1x2
```

- The ***female-male difference*** **in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

♦ ***<u>Test 5</u>:*** The ***female-male difference*** in conditional mean Y **does not depend on industry** – i.e., **industry effects** are ***equal*** for ***males*** **and** ***females***.

- The ***null*** **and** ***alternative*** **hypotheses** are:

  H_0: $\delta_6 = 0$ *and* $\delta_7 = 0$ *and* $\delta_8 = 0$

  H_1: $\delta_6 \neq 0$ *and/or* $\delta_7 \neq 0$ *and/or* $\delta_8 \neq 0$

- Compute an **F-test** of H_0 against H_1 using the following *Stata* **test** command:

```
test fin2 fin3 fin4
```

Evaluating the Marginal Effects of *Continuous* Explanatory Variables in Model 5.5

<u>Review of Model 5.5</u>: Quadratic terms in the continuous explanatory variables X_1 and X_2

The **population regression *equation* for Model 5.5** is:

$$\begin{aligned} Y_i = {} & \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i \\ & + \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i + u_i \end{aligned} \qquad \textbf{(5.5)}$$

The **population regression *function* for Model 5.5** is:

$$\begin{aligned} & E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i) \\ & = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i \\ & \quad + \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \end{aligned} \qquad \textbf{(5.5')}$$

***Stata* command** for computing OLS estimates of the pooled, full-interaction regression equation (5.5):

```
regress y x1 x2 x1sq x2sq x1x2 in2 in3 in4 f fx1 fx2 fx1sq fx2sq fx1x2 fin2
fin3 fin4
```

The **population regression *function* for Model 5.5** is:

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1}X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1}X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \quad \textbf{(5.5')}$$

- The ***female* population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 1$ in (5.5'):

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1}X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1}X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1)X_{i1} + (\beta_2 + \delta_2)X_{i2} + (\beta_3 + \delta_3)X_{i1}^2 + (\beta_4 + \delta_4)X_{i2}^2 + (\beta_5 + \delta_5)X_{i1}X_{i2}$$

$$+ (\beta_6 + \delta_6)IN2_i + (\beta_7 + \delta_7)IN3_i + (\beta_8 + \delta_8)IN4_i \quad \textbf{(5.5f)}$$

- The ***male* population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 0$ in (5.5'):

$$E(Y_i \mid F_i = 0, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1}X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

The Marginal Effect of X_1 in Model 5.5

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \qquad \textbf{(5.5')}$$

- The **marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial Y_i}{\partial X_{i1}} = \frac{\partial E\left(Y_i \mid F_i, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 F_i + 2\delta_3 F_i X_{i1} + \delta_5 F_i X_{i2}$$

- The **marginal effect of X_1 for *females* in Model 5.5** is obtained by setting $F_i = 1$:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=1} = \frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

$$= (\beta_1 + \delta_1) + 2(\beta_3 + \delta_3) X_{i1} + (\beta_5 + \delta_5) X_{i2}$$

- The **marginal effect of X_1 for *males* in Model 5.5** is obtained by setting $F_i = 0$:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=0} = \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2}$$

- The ***female-male difference* in the marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} - \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}}$$

$$= \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2} - \beta_1 - 2\beta_3 X_{i1} - \beta_5 X_{i2}$$

$$= \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

Evaluate the Marginal Effect of X_1 in Model 5.5 for Males and Females

- First, **select specific values of X_1 and X_2** at which to evaluate the **marginal effect of X_1 for *males* and *females* in Model 5.5**. Common choices for typical values of X_1 and X_2 are:

  (1) **sample *mean* values** of X_1 and X_2, denoted as $\bar{X}_1$ and $\bar{X}_2$;
  (2) **sample *median* values** of X_1 and X_2, denoted as $X_{1,50p}$ and $X_{2,50p}$.

  ***Stata* commands** for defining as scalars the **sample *median* values** of **X_1 and X_2:**

```
summarize x1, detail
return list
scalar x1med = r(p50)

summarize x2, detail
return list
scalar x2med = r(p50)

scalar list x1med x2med
```

- The **marginal effect of X_1 for *males* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=0} = \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2}$$

Evaluated at the **sample *median* values of X_1 and X_2**, the **marginal effect of X_1 for *males*** in Model 5.5 is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=0} = \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{1,50p} + \beta_5 X_{2,50p}$$

***Stata* command:**

```
lincom _b[x1] + 2*_b[x1sq]*x1med + _b[x1x2]*x2med
```

- The **marginal effect of X_1 for *females* in Model 5.5** is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=1} = \frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} = \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

$$= (\beta_1 + \delta_1) + 2(\beta_3 + \delta_3)X_{i1} + (\beta_5 + \delta_5)X_{i2}$$

Evaluated at the **sample *median* values of X_1 and X_2**, the **marginal effect of X_1 for *females*** in Model 5.5 is:

$$\left.\frac{\partial Y_i}{\partial X_{i1}}\right|_{F_i=1} = \frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} = (\beta_1 + \delta_1) + 2(\beta_3 + \delta_3)X_{1,50p} + (\beta_5 + \delta_5)X_{2,50p}$$

***Stata* command:**

```
lincom _b[x1] + _b[fx1] + 2*(_b[x1sq] + _b[fx1sq])*x1med + (_b[x1x2] +
_b[fx1x2])*x2med
```

- The ***female-male difference*** **in the marginal effect of X_1 in Model 5.5** is:

$$\frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} - \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}}$$

$$= \beta_1 + 2\beta_3 X_{i1} + \beta_5 X_{i2} + \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2} - \beta_1 - 2\beta_3 X_{i1} - \beta_5 X_{i2}$$

$$= \delta_1 + 2\delta_3 X_{i1} + \delta_5 X_{i2}$$

Evaluated at the **sample *median* values of X_1 and X_2**, the ***female-male*** **difference in the marginal effect of X_1** in Model 5.5 is:

$$\frac{\partial E\left(Y_i \mid F_i = 1, x_i^T\right)}{\partial X_{i1}} - \frac{\partial E\left(Y_i \mid F_i = 0, x_i^T\right)}{\partial X_{i1}} = \delta_1 + 2\delta_3 X_{1,50p} + \delta_5 X_{2,50p}$$

***Stata* command:**

```
lincom _b[fx1] + 2*_b[fx1sq])*x1med + _b[fx1x2])*x2med
```

Evaluating the *Marginal* Effects of the *Categorical* Explanatory Variable in Model 5.5

General Nature: The **marginal effects of a *categorical* explanatory variable such as industry** consist of **the differences in conditional mean values of Y between *pairs* of industry categories** – e.g., the **conditional mean Y difference between *males* in industries 4 and 2**, and the **conditional mean Y difference between *females* in industries 4 and 2.**

Recall that the **population regression *function* for Model 5.5** is:

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \qquad \textbf{(5.5')}$$

- The ***female* population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 1$ in (5.5'):

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1) X_{i1} + (\beta_2 + \delta_2) X_{i2} + (\beta_3 + \delta_3) X_{i1}^2 + (\beta_4 + \delta_4) X_{i2}^2 + (\beta_5 + \delta_5) X_{i1} X_{i2}$$

$$+ (\beta_6 + \delta_6) IN2_i + (\beta_7 + \delta_7) IN3_i + (\beta_8 + \delta_8) IN4_i \qquad \textbf{(5.5f)}$$

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \qquad \textbf{(5.5')}$$

- The ***male*** **population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 0$ in (5.5'):

$$E(Y_i \mid F_i = 0, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

- The ***female-male difference*** **in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

Marginal Effects of Industry for Males in Model 5.5

- The ***male* population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 0$ in (5.5'):

$$E(Y_i \mid F_i = 0, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

The ***Stata* command** for computing OLS estimates of Model 5.5 is:

```
regress y x1 x2 x1sq x2sq x1x2 in2 in3 in4 f fx1 fx2 fx1sq fx2sq fx1x2 fin2
fin3 fin4
```

1. The **industry 2-industry 1 difference** in **conditional mean Y for *males*** equals β_6 in Model 5.5. To display an estimate of β_6 in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in2]
```

2. The **industry 3-industry 1 difference** in **conditional mean Y for *males*** equals β_7 in Model 5.5. To display an estimate of β_7 in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in3]
```

- The ***male*** **population regression function for Model 5.5** is:

$$E(Y_i \mid F_i = 0, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

3. The **industry 4-industry 1 difference** in **conditional mean Y for *males*** equals β_8 in Model 5.5. To display an estimate of β_8 in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in4]
```

4. The **industry 3-industry 2 difference** in **conditional mean Y for *males*** equals $\beta_7 - \beta_6$ in Model 5.5. To compute an estimate of $\beta_7 - \beta_6$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in3] - _b[in2]
```

5. The **industry 4-industry 2 difference** in **conditional mean Y for *males*** equals $\beta_8 - \beta_6$ in Model 5.5. To compute an estimate of $\beta_8 - \beta_6$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in4] - _b[in2]
```

6. The **industry 4-industry 3 difference** in **conditional mean Y for *males*** equals $\beta_8 - \beta_7$ in Model 5.5. To compute an estimate of $\beta_8 - \beta_7$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in4] - _b[in3]
```

Marginal **Effects of Industry for Females in Model 5.5**

- The ***female*** **population regression function for Model 5.5** is:

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1)X_{i1} + (\beta_2 + \delta_2)X_{i2} + (\beta_3 + \delta_3)X_{i1}^2 + (\beta_4 + \delta_4)X_{i2}^2 + (\beta_5 + \delta_5)X_{i1}X_{i2}$$
$$+ (\beta_6 + \delta_6)IN2_i + (\beta_7 + \delta_7)IN3_i + (\beta_8 + \delta_8)IN4_i \qquad \textbf{(5.5f)}$$

Again, the ***Stata*** **command** for computing OLS estimates of regression equation (5.5) is:

```
regress y x1 x2 x1sq x2sq x1x2 in2 in3 in4 f fx1 fx2 fx1sq fx2sq fx1x2 fin2
fin3 fin4
```

1. The **industry 2-industry 1 difference** in **conditional mean Y for *females*** equals $\boldsymbol{\beta_6 + \delta_6}$ in Model 5.5. To compute an estimate of $(\beta_6 + \delta_6)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in2] + _b[fin2]
```

2. The **industry 3-industry 1 difference** in **conditional mean Y for *females*** equals $\boldsymbol{\beta_7 + \delta_7}$ in Model 5.5. To compute an estimate of $(\beta_7 + \delta_7)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in3] + _b[fin3]
```

- The ***female*** **population regression function for Model 5.5** is:

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1)X_{i1} + (\beta_2 + \delta_2)X_{i2} + (\beta_3 + \delta_3)X_{i1}^2 + (\beta_4 + \delta_4)X_{i2}^2 + (\beta_5 + \delta_5)X_{i1}X_{i2}$$
$$+ (\beta_6 + \delta_6)IN2_i + (\beta_7 + \delta_7)IN3_i + (\beta_8 + \delta_8)IN4_i \quad \textbf{(5.5f)}$$

3. The **industry 4-industry 1 difference** in **conditional mean Y for *females*** equals $\boldsymbol{\beta_8 + \delta_8}$ in Model 5.5. To compute an estimate of $(\beta_8 + \delta_8)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in4] + _b[fin4]
```

4. The **industry 3-industry 2 difference** in **conditional mean Y for *females*** equals $\boldsymbol{(\beta_7 + \delta_7) - (\beta_6 + \delta_6)}$ in Model 5.5. To compute an estimate of $(\beta_7 + \delta_7) - (\beta_6 + \delta_6)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in3] + _b[fin3] – (_b[in2] + _b[fin2])
```

5. The **industry 4-industry 2 difference** in **conditional mean Y for *females*** equals $\boldsymbol{(\beta_8 + \delta_8) - (\beta_6 + \delta_6)}$ in Model 5.5. To compute an estimate of $(\beta_8 + \delta_8) - (\beta_6 + \delta_6)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in4] + _b[fin4] – (_b[in2] + _b[fin2])
```

- The ***female* population regression function for Model 5.5** is:

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1)X_{i1} + (\beta_2 + \delta_2)X_{i2} + (\beta_3 + \delta_3)X_{i1}^2 + (\beta_4 + \delta_4)X_{i2}^2 + (\beta_5 + \delta_5)X_{i1}X_{i2}$$
$$+ (\beta_6 + \delta_6)IN2_i + (\beta_7 + \delta_7)IN3_i + (\beta_8 + \delta_8)IN4_i \qquad \textbf{(5.5f)}$$

6. The **industry 4-industry 3 difference** in **conditional mean Y for *females*** equals $\mathbf{(\beta_8 + \delta_8) - (\beta_7 + \delta_7)}$ in Model 5.5. To compute an estimate of $(\beta_8 + \delta_8) - (\beta_7 + \delta_7)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[in4] + _b[fin4] – (_b[in3] + _b[fin3])
```

Female-Male Differences in the *Marginal* Effects of Industry in Model 5.5

- The ***female-male difference*** **in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

Again, the ***Stata* command** for computing OLS estimates of regression equation (5.5) is:

```
regress y x1 x2 x1sq x2sq x1x2 in2 in3 in4 f fx1 fx2 fx1sq fx2sq fx1x2 fin2
fin3 fin4
```

1. The **industry 2-industry 1 difference** in **conditional mean Y for *females*** equals $\boldsymbol{\beta_6 + \delta_6}$ in Model 5.5. The **industry 2-industry 1 difference** in **conditional mean Y for *males*** equals $\boldsymbol{\beta_6}$.

   The **female-male difference** in the **industry 2-industry 1 difference** in **conditional mean Y**

   $$= \boldsymbol{\beta_6 + \delta_6 - \beta_6 = \delta_6}$$

   To display an estimate of δ_6 in Model 5.5, use the following *Stata* **limcom** command:

   ```
   lincom _b[fin2]
   ```

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

2. The **industry 3-industry 1 difference** in **conditional mean Y for *females*** equals $\boldsymbol{\beta_7 + \delta_7}$ in Model 5.5. The **industry 3-industry 1 difference** in **conditional mean Y for *males*** equals $\boldsymbol{\beta_7}$.

   The **female-male difference** in the **industry 3-industry 1 difference** in **conditional mean Y**

   $= \boldsymbol{\beta_7 + \delta_7 - \beta_7 = \delta_7}$

   To display an estimate of δ_7 in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[fin3]
```

3. The **industry 4-industry 1 difference** in **conditional mean Y for *females*** equals $\boldsymbol{\beta_8 + \delta_8}$ in Model 5.5. The **industry 4-industry 1 difference** in **conditional mean Y for *males*** equals $\boldsymbol{\beta_8}$.

   The **female-male difference** in the **industry 4-industry 1 difference** in **conditional mean Y**

   $= \boldsymbol{\beta_8 + \delta_8 - \beta_8 = \delta_8}$

   To display an estimate of δ_8 in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[fin4]
```

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

4. The **industry 3-industry 2 difference** in **conditional mean Y for *females*** equals $(\beta_7 + \delta_7) - (\beta_6 + \delta_6)$ in Model 5.5. The **industry 3-industry 2 difference** in **conditional mean Y for *males*** equals $\beta_7 - \beta_6$.

   The **female-male difference** in the **industry 3-industry 2 difference** in **conditional mean Y**

   $$= (\beta_7 + \delta_7) - (\beta_6 + \delta_6) - (\beta_7 - \beta_6)$$

   $$= \beta_7 + \delta_7 - \beta_6 - \delta_6 - (\beta_7 - \beta_6)$$

   $$= (\beta_7 - \beta_6) + (\delta_7 - \delta_6) - (\beta_7 - \beta_6)$$

   $$= (\delta_7 - \delta_6)$$

   To compute an estimate of $(\delta_7 - \delta_6)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[fin3] – _b[fin2]
```

- The ***female-male difference*** **in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

5. The **industry 4-industry 2 difference** in **conditional mean Y for *females*** equals $(\beta_8 + \delta_8) - (\beta_6 + \delta_6)$ in Model 5.5. The **industry 4-industry 2 difference** in **conditional mean Y for *males*** equals $\beta_8 - \beta_6$.

   The **female-male difference** in the **industry 4-industry 2 difference** in **conditional mean Y**

   $$= (\beta_8 + \delta_8) - (\beta_6 + \delta_6) - (\beta_8 - \beta_6)$$

   $$= \beta_8 + \delta_8 - \beta_6 - \delta_6 - (\beta_8 - \beta_6)$$

   $$= (\beta_8 - \beta_6) + (\delta_8 - \delta_6) - (\beta_8 - \beta_6)$$

   $$= (\delta_8 - \delta_6)$$

   To compute an estimate of $(\delta_8 - \delta_6)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[fin4] – _b[fin2]
```

- The ***female-male difference* in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

6. The **industry 4-industry 3 difference** in **conditional mean Y for *females*** equals $(\boldsymbol{\beta_8 + \delta_8}) - (\boldsymbol{\beta_7 + \delta_7})$ in Model 5.5. The **industry 4-industry 3 difference** in **conditional mean Y for *males*** equals $\boldsymbol{\beta_8 - \beta_7}$.

The **female-male difference** in the **industry 4-industry 2 difference** in **conditional mean Y**

$$= (\beta_8 + \delta_8) - (\beta_7 + \delta_7) - (\beta_8 - \beta_7)$$
$$= \beta_8 + \delta_8 - \beta_7 - \delta_7 - (\beta_8 - \beta_7)$$
$$= (\beta_8 - \beta_7) + (\delta_8 - \delta_7) - (\beta_8 - \beta_7)$$
$$= (\delta_8 - \delta_7)$$

To compute an estimate of $(\delta_8 - \delta_7)$ in Model 5.5, use the following *Stata* **limcom** command:

```
lincom _b[fin4] – _b[fin3]
```

Evaluating the *Conditional* Effects of the *Categorical* Explanatory Variable in Model 5.5

General Nature: The **conditional effects of a *categorical* explanatory variable such as industry** consist of **the conditional mean value of Y for each of the four industry categories** – industry 1, industry 2, industry 3, and industry 4 – for pre-selected values of the other explanatory variables in the model, such as the continuous explanatory variables X_1 and X_2 in Model 5.5.

Recall that the **population regression *function* for Model 5.5** is:

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \quad \textbf{(5.5')}$$

- The ***female* population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 1$ in (5.5'):

$$E(Y_i \mid F_i = 1, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

$$= (\beta_0 + \delta_0) + (\beta_1 + \delta_1) X_{i1} + (\beta_2 + \delta_2) X_{i2} + (\beta_3 + \delta_3) X_{i1}^2 + (\beta_4 + \delta_4) X_{i2}^2 + (\beta_5 + \delta_5) X_{i1} X_{i2}$$

$$+ (\beta_6 + \delta_6) IN2_i + (\beta_7 + \delta_7) IN3_i + (\beta_8 + \delta_8) IN4_i \quad \textbf{(5.5f)}$$

$$E(Y_i \mid X_{i1}, X_{i2}, F_i, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

$$+ \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i \qquad \textbf{(5.5')}$$

- The ***male*** **population regression function for Model 5.5** is obtained by setting the female indicator $F_i = 0$ in (5.5'):

$$E(Y_i \mid F_i = 0, X_{i1}, X_{i2}, IN2_i, IN3_i, IN4_i)$$

$$= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i$$

- The ***female-male difference*** **in conditional mean Y for Model 5.5** is:

$$E(Y_i \mid F_i = 1, x_i^T) - E(Y_i \mid F_i = 0, x_i^T)$$

$$= \delta_0 + \delta_1 X_{i1} + \delta_2 X_{i2} + \delta_3 X_{i1}^2 + \delta_4 X_{i2}^2 + \delta_5 X_{i1} X_{i2} + \delta_6 IN2_i + \delta_7 IN3_i + \delta_8 IN4_i$$

<u>Objective</u>: To compute estimates of the ***conditional*** **effect of industry** for **females and males**, and of the **female-male difference** in the conditional effect of industry **for *pre-selected values* of all the other explanatory variables** in the model.

- First, **select the values of X_1 and X_2** at which to evaluate the **conditional effect of industry for *females* and *males* in Model 5.5**.

  Suppose we select the ***pooled*** **sample means** of the continuous explanatory variables X_1 and X_2, which are denoted as $\bar{X}_1$ and $\bar{X}_2$.

  Stata **commands:**

```
summarize x1, detail
return list
scalar x1bar = r(mean)

summarize x2, detail
return list
scalar x2bar = r(mean)

scalar list x1bar x2bar
```

- Second, estimate by OLS the pooled, full-interaction regression equation (5.5):

$$
\begin{aligned}
Y_i = {} & \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \beta_6 IN2_i + \beta_7 IN3_i + \beta_8 IN4_i \\
& + \delta_0 F_i + \delta_1 F_i X_{i1} + \delta_2 F_i X_{i2} + \delta_3 F_i X_{i1}^2 + \delta_4 F_i X_{i2}^2 + \delta_5 F_i X_{i1} X_{i2} + \delta_6 F_i IN2_i + \delta_7 F_i IN3_i + \delta_8 F_i IN4_i + u_i \qquad \textbf{(5.5)}
\end{aligned}
$$

The ***Stata* command** for computing OLS estimates of regression equation (5.5) is:

```
regress y x1 x2 x1sq x2sq x1x2 in2 in3 in4 f fx1 fx2 fx1sq fx2sq fx1x2 fin2
fin3 fin4
```

- **For *females***, evaluate the ***female* sample regression function** for each of the **four *industry* categories** at the pre-selected values of the other explanatory variables X_1 and X_2, i.e., at the pooled sample means $\overline{X}_1$ and $\overline{X}_2$.

$$\hat{E}(Y_i \mid F_i = 1, \overline{X}_1, \overline{X}_2, IN2_i, IN3_i, IN4_i)$$

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2$$
$$+ (\hat{\beta}_6 + \hat{\delta}_6)IN2_i + (\hat{\beta}_7 + \hat{\delta}_7)IN3_i + (\hat{\beta}_8 + \hat{\delta}_8)IN4_i$$

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2$$

for industry 1 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 0$)

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2 + (\hat{\beta}_6 + \hat{\delta}_6)$$

for industry 2 ($IN2_i = 1$, $IN3_i = 0$, $IN4_i = 0$)

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2 + (\hat{\beta}_7 + \hat{\delta}_7)$$

for industry 3 ($IN2_i = 0$, $IN3_i = 1$, $IN4_i = 0$)

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2 + (\hat{\beta}_8 + \hat{\delta}_8)$$

for industry 4 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 1$)

***Stata* commands for *females*:**

To compute $\hat{E}(Y_i \mid F_i = 1, \overline{X}_1, \overline{X}_2, IN2_i = 0, IN3_i = 0, IN4_i = 0)$

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2$$

for industry 1 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 0$)

```
lincom _b[_cons] + _b[f] + (_b[x1] + _b[fx1])*x1bar + (_b[x2]
+_b[fx2])*x2bar + (_b[x1sq] + _b[fx1sq])*x1bar*x1bar + (_b[x2sq] +
_b[fx2sq])*x2bar*x2bar + (_b[x1x2] + _b[fx1x2])*x1bar*x2bar
```

To compute $\hat{E}(Y_i \mid F_i = 1, \overline{X}_1, \overline{X}_2, IN2_i = 1, IN3_i = 0, IN4_i = 0)$

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2 + (\hat{\beta}_6 + \hat{\delta}_6)$$

for industry 2 ($IN2_i = 1$, $IN3_i = 0$, $IN4_i = 0$)

```
lincom _b[_cons] + _b[f] + (_b[x1] + _b[fx1])*x1bar + (_b[x2] +
_b[fx2])*x2bar + (_b[x1sq] + _b[fx1sq])*x1bar*x1bar + (_b[x2sq] +
_b[fx2sq])*x2bar*x2bar + (_b[x1x2] + _b[fx1x2])*x1bar*x2bar + (_b[in2] +
_b[fin2])
```

To compute $\hat{E}(Y_i \mid F_i = 1, \overline{X}_1, \overline{X}_2, IN2_i = 0, IN3_i = 1, IN4_i = 0)$

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2 + (\hat{\beta}_7 + \hat{\delta}_7)$$

for industry 3 ($IN2_i = 0$, $IN3_i = 1$, $IN4_i = 0$)

```
lincom _b[_cons] + _b[f] + (_b[x1] + _b[fx1])*x1bar + (_b[x2] +
_b[fx2])*x2bar + (_b[x1sq] + _b[fx1sq])*x1bar*x1bar + (_b[x2sq] +
_b[fx2sq])*x2bar*x2bar + (_b[x1x2] + _b[fx1x2])*x1bar*x2bar + (_b[in3] +
_b[fin3])
```

To compute $\hat{E}(Y_i \mid F_i = 1, \overline{X}_1, \overline{X}_2, IN2_i = 0, IN3_i = 0, IN4_i = 1)$

$$= (\hat{\beta}_0 + \hat{\delta}_0) + (\hat{\beta}_1 + \hat{\delta}_1)\overline{X}_1 + (\hat{\beta}_2 + \hat{\delta}_2)\overline{X}_2 + (\hat{\beta}_3 + \hat{\delta}_3)\overline{X}_1^2 + (\hat{\beta}_4 + \hat{\delta}_4)\overline{X}_2^2 + (\hat{\beta}_5 + \hat{\delta}_5)\overline{X}_1\overline{X}_2 + (\hat{\beta}_8 + \hat{\delta}_8)$$

for industry 4 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 1$)

```
lincom _b[_cons] + _b[f] + (_b[x1] + _b[fx1])*x1bar + (_b[x2] +
_b[fx2])*x2bar + (_b[x1sq] + _b[fx1sq])*x1bar*x1bar + (_b[x2sq] +
_b[fx2sq])*x2bar*x2bar + (_b[x1x2] + _b[fx1x2])*x1bar*x2bar + (_b[in4] +
_b[fin4])
```

- **For *males***, evaluate the ***male* sample regression function** for each of the **four *industry* categories** at the pre-selected values of the other explanatory variables X_1 and X_2, i.e., at the pooled sample means $\overline{X}_1$ and $\overline{X}_2$.

$$\hat{E}(Y_i \mid F_i = 0, \overline{X}_1, \overline{X}_2, IN2_i, IN3_i, IN4_i)$$

$$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2 + \hat{\beta}_6 IN2_i + \hat{\beta}_7 IN3_i + \hat{\beta}_8 IN4_i$$

$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2$ for industry 1 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 0$)

$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2 + \hat{\beta}_6$ for industry 2 ($IN2_i = 1$, $IN3_i = 0$, $IN4_i = 0$)

$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2 + \hat{\beta}_7$ for industry 3 ($IN2_i = 0$, $IN3_i = 1$, $IN4_i = 0$)

$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2 + \hat{\beta}_8$ for industry 4 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 1$)

***Stata* commands for *males*:**

To compute $\hat{E}(Y_i \mid F_i = 0, \overline{X}_1, \overline{X}_2, IN2_i = 0, IN3_i = 0, IN4_i = 0)$

$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2$ for industry 1 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 0$)

```
lincom _b[_cons] + _b[x1]*x1bar + _b[x2]*x2bar + _b[x1sq]*x1bar*x1bar +
_b[x2sq]*x2bar*x2bar + _b[x1x2]*x1bar*x2bar
```

To compute $\hat{E}(Y_i \mid F_i = 0, \overline{X}_1, \overline{X}_2, IN2_i = 1, IN3_i = 0, IN4_i = 0)$

$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2 + \hat{\beta}_6$ for industry 2 ($IN2_i = 1$, $IN3_i = 0$, $IN4_i = 0$)

```
lincom _b[_cons] + _b[x1]*x1bar + _b[x2]*x2bar + _b[x1sq]*x1bar*x1bar +
_b[x2sq]*x2bar*x2bar + _b[x1x2]*x1bar*x2bar + _b[in2]
```

To compute $\hat{E}(Y_i \mid F_i = 0, \overline{X}_1, \overline{X}_2, IN2_i = 0, IN3_i = 1, IN4_i = 0)$

$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2 + \hat{\beta}_7$ for industry 3 ($IN2_i = 0$, $IN3_i = 1$, $IN4_i = 0$)

```
lincom _b[_cons] + _b[x1]*x1bar + _b[x2]*x2bar + _b[x1sq]*x1bar*x1bar +
_b[x2sq]*x2bar*x2bar + _b[x1x2]*x1bar*x2bar + _b[in3]
```

To compute $\hat{E}(Y_i \mid F_i = 0, \overline{X}_1, \overline{X}_2, IN2_i = 0, IN3_i = 0, IN4_i = 1)$

$= \hat{\beta}_0 + \hat{\beta}_1\overline{X}_1 + \hat{\beta}_2\overline{X}_2 + \hat{\beta}_3\overline{X}_1^2 + \hat{\beta}_4\overline{X}_2^2 + \hat{\beta}_5\overline{X}_1\overline{X}_2 + \hat{\beta}_8$ for industry 4 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 1$)

```
lincom _b[_cons] + _b[x1]*x1bar + _b[x2]*x2bar + _b[x1sq]*x1bar*x1bar +
_b[x2sq]*x2bar*x2bar + _b[x1x2]*x1bar*x2bar + _b[in4]
```

- **For the *female-male differences*** in the **conditional effect of industry**, evaluate the **female-male difference in sample regression functions** for each of the four industry categories at the pre-selected values of the other explanatory variables X_1 and X_2, i.e., at the pooled sample means $\overline{X}_1$ and $\overline{X}_2$.

$$\hat{E}(Y_i \mid F_i = 1, \bar{x}^T, IN2_i, IN3_i, IN4_i) - \hat{E}(Y_i \mid F_i = 0, \bar{x}^T, IN2_i, IN3_i, IN4_i)$$

$$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2 + \hat{\delta}_6 IN2_i + \hat{\delta}_7 IN3_i + \hat{\delta}_8 IN4_i$$

$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2$ for industry 1 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 0$)

$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2 + \hat{\delta}_6$ for industry 2 ($IN2_i = 1$, $IN3_i = 0$, $IN4_i = 0$)

$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2 + \hat{\delta}_7$ for industry 3 ($IN2_i = 0$, $IN3_i = 1$, $IN4_i = 0$)

$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2 + \hat{\delta}_8$ for industry 4 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 1$)

***Stata* commands for *female-male differences*:**

To compute $\hat{E}(Y_i \mid F_i = 1, \bar{x}^T, IN1_i = 1) - \hat{E}(Y_i \mid F_i = 0, \bar{x}^T, IN1_i = 1)$

$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2$ for industry 1 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 0$)

```
lincom _b[f] + _b[fx1]*x1bar + _b[fx2]*x2bar + _b[fx1sq]*x1bar*x1bar +
_b[fx2sq]*x2bar*x2bar + _b[fx1x2]*x1bar*x2bar
```

To compute $\hat{E}(Y_i \mid F_i = 1, \bar{x}^T, IN2_i = 1) - \hat{E}(Y_i \mid F_i = 0, \bar{x}^T, IN2_i = 1)$

$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2 + \hat{\delta}_6$ for industry 2 ($IN2_i = 1$, $IN3_i = 0$, $IN4_i = 0$)

```
lincom _b[f] + _b[fx1]*x1bar + _b[fx2]*x2bar + _b[fx1sq]*x1bar*x1bar +
_b[fx2sq]*x2bar*x2bar + _b[fx1x2]*x1bar*x2bar + _b[fin2]
```

To compute $\hat{E}(Y_i \mid F_i = 1, \bar{x}^T, IN3_i = 1) - \hat{E}(Y_i \mid F_i = 0, \bar{x}^T, IN3_i = 1)$

$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2 + \hat{\delta}_7$ for industry 3 ($IN2_i = 0$, $IN3_i = 1$, $IN4_i = 0$)

```
lincom _b[f] + _b[fx1]*x1bar + _b[fx2]*x2bar + _b[fx1sq]*x1bar*x1bar +
_b[fx2sq]*x2bar*x2bar + _b[fx1x2]*x1bar*x2bar + _b[fin3]
```

To compute $\hat{E}(Y_i \mid F_i = 1, \bar{x}^T, IN4_i = 1) - \hat{E}(Y_i \mid F_i = 0, \bar{x}^T, IN4_i = 1)$

$= \hat{\delta}_0 + \hat{\delta}_1\overline{X}_1 + \hat{\delta}_2\overline{X}_2 + \hat{\delta}_3\overline{X}_1^2 + \hat{\delta}_4\overline{X}_2^2 + \hat{\delta}_5\overline{X}_1\overline{X}_2 + \hat{\delta}_8$ for industry 4 ($IN2_i = 0$, $IN3_i = 0$, $IN4_i = 1$)

```
lincom _b[f] + _b[fx1]*x1bar + _b[fx2]*x2bar + _b[fx1sq]*x1bar*x1bar +
_b[fx2sq]*x2bar*x2bar + _b[fx1x2]*x1bar*x2bar + _b[fin4]
```