ECON 452* -- Important Matrix Formulas from Notes 9 and 10

Important Matrix Formulas for Classical Linear Regression Model

- **The population regression equation (PRE):**

$y = X\beta + u$ by assumption A1

$E(u \mid X) = \underline{0}$ by assumption A2: zero conditional mean errors

$V(u \mid X) = \sigma^2 I_N$ by assumption A3: spherical errors

- **OLS estimator of coefficient vector β:**

$$\hat{\beta}_{OLS} = \left(X^T X\right)^{-1} X^T y$$

- **OLS estimation criterion:**

$$\underset{\{\hat{\beta}\}}{\text{Minimize}}\ RSS(\hat{\beta}) = \hat{u}^T\hat{u} = \left(y - X\hat{\beta}\right)^T \left(y - X\hat{\beta}\right) = y^T y - 2\hat{\beta}^T X^T y + \hat{\beta}^T X^T X \hat{\beta}$$

- **OLS normal equations:**

$$X^T X \hat{\beta}_{OLS} = X^T y$$

- **OLS sample regression equation (SRE):**

$$y = X\hat{\beta} + \hat{u} = \hat{y} + \hat{u}$$

where

$$\hat{y} = X\hat{\beta}$$

$$\hat{u} = y - \hat{y} = y - X\hat{\beta}$$

- **Sampling distribution of $\hat{\beta}_{OLS}$:**

Mean of $\hat{\beta}_{OLS}$: depends on assumptions A1 and A2

$E(\hat{\beta}_{OLS}|X) = E(\hat{\beta}_{OLS}) = \beta \;\Rightarrow\; \hat{\beta}_{OLS}$ **is an *unbiased* estimator of β**

Variance-Covariance Matrix of $\hat{\beta}_{OLS}$: depends on assumptions A1, A2, A3

$$V(\hat{\beta}_{OLS}|X) = V(\hat{\beta}_{OLS}) = \sigma^2 (X^T X)^{-1}$$

- **Unbiased estimator of σ^2:**

$$\hat{\sigma}^2_{OLS} = \frac{RSS(\hat{\beta}_{OLS})}{N-K} = \frac{\hat{u}^T\hat{u}}{N-K}$$

where

$$RSS(\hat{\beta}_{OLS}) = \hat{u}^T\hat{u} = (y - X\hat{\beta}_{OLS})^T (y - X\hat{\beta}_{OLS})$$

$=$ the residual sum of squares for the coefficient estimator $\hat{\beta}_{OLS}$

- **Unbiased estimator of $V(\hat{\beta}_{OLS}|X)$:**

$$\hat{V}(\hat{\beta}_{OLS}|X) = \hat{V}(\hat{\beta}_{OLS}) = \hat{V}_{OLS} = \hat{\sigma}^2_{OLS}(X^T X)^{-1}$$

- **Test of q linear equality restrictions on the coefficient vector β:**

H_0: $R\beta = r \;\Leftrightarrow\; R\beta - r = \underline{0}$

H_1: $R\beta \neq r \;\Leftrightarrow\; R\beta - r \neq \underline{0}$

- **OLS Wald F-statistic:**

$$F_W = \frac{1}{q} W_{OLS} = \frac{(R\hat{\beta} - r)^T (R\hat{V}_{OLS} R^T)^{-1} (R\hat{\beta} - r)}{q} \sim F[q, N-K] \text{ under } H_0$$