be less than $2k+1$. The easiest way to see why this will almost always be the case is to consider an example.

Suppose that the regression function $x_t(\boldsymbol{\beta})$ for the original H_0 model is

$$\beta_0 + \beta_1 z_t + \beta_2 t + \beta_3 z_{t-1} + \beta_4 y_{t-1}, \tag{10.93}$$

where z_t is the t^{th} observation on an economic time series, and t is the t^{th} observation on a linear time trend. The regression function for the unrestricted H_2 model which corresponds to (10.93) is

$$\begin{aligned} &\beta_0 + \beta_1 z_t + \beta_2 t + \beta_3 z_{t-1} + \beta_4 y_{t-1} + \rho y_{t-1} \\ &\quad + \gamma_0 + \gamma_1 z_{t-1} + \gamma_2 (t-1) + \gamma_3 z_{t-2} + \gamma_4 y_{t-2}. \end{aligned} \tag{10.94}$$

This regression function appears to have 11 parameters, but 4 of them are in fact unidentifiable. It is obvious that we cannot estimate both β_0 and γ_0, since there cannot be two constant terms. Similarly, we cannot estimate both β_3 and γ_1, since there cannot be two coefficients on z_{t-1}, and we cannot estimate both β_4 and ρ, since there cannot be two coefficients on y_{t-1}. We also cannot estimate γ_2 along with β_2 and the constant, because t, $t-1$ and the constant term are perfectly collinear, since $t-(t-1)=1$. Thus the version of H_2 that can actually be estimated has the regression function

$$\delta_0 + \beta_1 z_t + \delta_1 t + \delta_2 z_{t-1} + \delta_3 y_{t-1} + \gamma_3 z_{t-2} + \gamma_4 y_{t-2}, \tag{10.95}$$

where

$$\delta_0 = \beta_0 + \gamma_0 - \gamma_2;\;\; \delta_1 = \beta_2 + \gamma_2;\;\; \delta_2 = \beta_3 + \gamma_1;\; \text{and } \delta_3 = \rho + \beta_4.$$

We see that (10.95) has seven identifiable parameters: β_1, γ_3, γ_4, and δ_0 through δ_3, instead of the eleven parameters, many of them not identifiable, of (10.94). The regression function for the restricted model, H_1, is

$$\begin{aligned} &\beta_0 + \beta_1 z_t + \beta_2 t + \beta_3 z_{t-1} + \beta_4 y_{t-1} + \rho y_{t-1} \\ &\quad - \rho\beta_0 - \rho\beta_1 z_{t-1} - \rho\beta_2 (t-1) - \rho\beta_3 z_{t-2} - \rho\beta_4 y_{t-2}, \end{aligned}$$

and it has six parameters, ρ and β_0 through β_4. Thus, in this case, l, the number of restrictions that H_1 imposes on H_2, is just 1.

While this is a slightly extreme example, similar problems arise in almost every attempt to test common factor restrictions. Constant terms, many types of dummy variables (notably seasonal dummies and time trends), lagged dependent variables, and independent variables that appear with more than one time subscript almost always result in an unrestricted model H_2 of which not all parameters will be identifiable. Luckily, it is very easy to deal with these problems when one does an F test; one simply has to omit the redundant regressors when estimating H_2. One can then calculate l as the number of parameters in H_2 minus the number in H_1, which is $k+1$. Since many regression packages automatically drop redundant regressors, one naive but often effective approach is simply to attempt to estimate H_2 in something close to its original form and then to count the number of parameters that the regression package is actually able to estimate.

The F test (10.92) is not the only way to test common factor restrictions. Since the regression function for H_2 is linear in all parameters, while the one for H_1 is nonlinear, it is natural to try to base tests on the OLS estimates of H_2 alone. One approach to this problem is discussed by Sargan (1980a), but it is quite complicated and requires specialized computer software. A simpler approach is to use a one-step estimator of H_1. Consistent estimates of the parameters of H_1 may be obtained from the estimates of H_2, as discussed in Section 10.3, and the GNR (10.19) is then used to obtain one-step estimates. These estimates themselves are not necessarily of interest. All that is needed is the sum of squared residuals from the GNR, which may be used in place of SSR_1 in the formula (10.92) for the F test. However, since it is generally neither difficult nor expensive to estimate H_1 with modern computers and software packages, situations in which there is a significant advantage from the use of this one-step procedure are likely to be rare.

Something very like a test of common factor restrictions can be employed even when the original (H_0) model is nonlinear. In this case, the H_1 model can be written as

$$(1-\rho L)y_t = (1-\rho L)x_t(\boldsymbol{\beta}) + \varepsilon_t. \tag{10.96}$$

A version of (10.96) in which the common factor restriction does not hold is

$$(1-\rho L)y_t = (1-\delta L)x_t(\boldsymbol{\beta}) + \varepsilon_t. \tag{10.97}$$

Evidently, (10.96) is just (10.97) subject to the restriction that $\delta = \rho$. This restriction can be tested by a Gauss-Newton regression in the usual way. This GNR is

$$\begin{aligned}\boldsymbol{y} - \hat{\boldsymbol{x}} - \hat\rho(\boldsymbol{y}_{-1} - \hat{\boldsymbol{x}}_{-1}) = (\hat{\boldsymbol{X}} - \hat\rho\hat{\boldsymbol{X}}_{-1})\boldsymbol{b}& \\ + r(\boldsymbol{y}_{-1} - \hat{\boldsymbol{x}}_{-1}) + d\hat{\boldsymbol{x}}_{-1} + \text{residuals},&\end{aligned} \tag{10.98}$$

where $\hat\rho$ and $\hat{\boldsymbol{\beta}}$ are the NLS estimates of H_1, and $\hat{\boldsymbol{x}} \equiv \boldsymbol{x}(\hat{\boldsymbol{\beta}})$. Regression (10.98) looks exactly like the GNR (10.26), which we used to calculate the covariance matrix of $\hat{\boldsymbol{\beta}}$ and $\hat\rho$, with the addition of the extra regressor $\hat{\boldsymbol{x}}_{-1}$, the coefficient of which is d. The t statistic for $d = 0$ will be an asymptotically valid test statistic.

Notice that this GNR could be used even if $x_t(\boldsymbol{\beta})$ were a linear function. Since this variant of the common factor restrictions test necessarily has only one degree of freedom, it would not be the same as the usual form of the test, discussed above, for any model with $l > 1$. The difference arises because