

on asymptotic theory, and one cannot hope to obtain consistent parameter estimates if the parameters are not asymptotically identified.

In this section, we will discuss the asymptotic identifiability of a linear simultaneous equations model by the two-stage least squares estimator introduced in Section 7.5. This may seem a very limited topic, and in a certain sense it is indeed limited. However, it is a topic that has given rise to a truly vast literature, to which we can in no way do justice here; see Fisher (1976) and Hsiao (1983). There exist models that are not identified by the 2SLS estimator but are identified by other estimators, such as the FIML estimator, and we will briefly touch on such cases later. It is not a simple task to extend the theory we will present in this section to the context of nonlinear models, for which it is usually better to return to the general theory expounded in Section 5.2.

We begin with the linear simultaneous equations model, (18.01). This model consists of DGPs that generate samples for which each observation is a  $g$ -vector  $\mathbf{Y}_t$  of dependent variables, conditional on a set of exogenous and lagged dependent variables  $\mathbf{X}_t$ . Since the exogenous variables in  $\mathbf{X}_t$  are assumed to be weakly exogenous, their generating mechanism can be ignored. In order to discuss identification, little needs to be assumed about the error terms  $\mathbf{U}_t$ . They must evidently satisfy the condition that  $E(\mathbf{U}_t) = \mathbf{0}$ , and it seems reasonable to assume that they are serially independent and that  $E(\mathbf{U}_t^\top \mathbf{U}_t) = \boldsymbol{\Sigma}_t$ , where  $\boldsymbol{\Sigma}_t$  is a positive definite matrix for all  $t$ . If inferences are to be based on the usual 2SLS covariance matrix, it will be necessary to make the further assumption that the error terms are homoskedastic, that is,  $\boldsymbol{\Sigma}_t = \boldsymbol{\Sigma}$  for all  $t$ .

It is convenient to treat the identification of the parameters of a simultaneous equations model equation by equation, since it is entirely possible that the parameters of some equations may be identified while the parameters of others are not. In order to simplify notation, we will consider, without loss of generality, only the parameters of the first equation of the system, that is, the elements of the first columns of the matrices  $\boldsymbol{\Gamma}$  and  $\mathbf{B}$ . As we remarked in Section 18.1, restrictions must be imposed on the elements of these matrices for identification to be possible. It is usual to assume that these restrictions all take the form of zero restrictions on some elements. A variable is said to be **excluded** from an equation if the coefficient corresponding to that variable for that equation is restricted to be zero; otherwise, it is said to be **included** in the equation. As discussed in Section 6.4, it is always possible in the context of a single equation to perform a reparametrization such that all restrictions take the form of zero restrictions. But in the context of a simultaneous equations model, such reparametrizations exist in general only if there are no **cross-equation restrictions**, that is, restrictions which involve the parameters of more than one equation of the system. If there are cross-equation restrictions, then to all intents and purposes we leave the context of linear systems. We would in any case have to abandon the 2SLS estimator if we wished to impose cross-equation restrictions.