unidentified. However, following the procedure used to obtain the J and P tests, we can replace the parameters of the model that is *not* being tested by estimates. Thus, if we wish to test H_1, we can replace $\boldsymbol{\gamma}$ and σ_2 by ML estimates $\hat{\boldsymbol{\gamma}}$ and $\hat{\sigma}_2$ so that H_C becomes

$$H_C'\colon\ (1-\alpha)\left(\frac{y_t - x_t(\boldsymbol{\beta})}{\sigma_1}\right) + \alpha\left(\frac{\log y_t - z_t(\hat{\boldsymbol{\gamma}})}{\hat{\sigma}_2}\right) = \varepsilon_t.$$

It is straightforward to test H_1 against H_C' by means of the DLR:

$$\begin{bmatrix} \dfrac{(y_t - \hat{x}_t)}{\hat{\sigma}_1} \\ 1 \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{X}}_t & \dfrac{(y_t - \hat{x}_t)}{\hat{\sigma}_1} & \hat{z}_t - \log y_t \\ \mathbf{0} & -1 & \hat{\sigma}_1 / y_t \end{bmatrix} \begin{bmatrix} \boldsymbol{b} \\ s \\ a \end{bmatrix} + \text{residuals}, \tag{14.45}$$

where $\hat{x}_t \equiv x_t(\hat{\boldsymbol{\beta}})$, $\hat{\boldsymbol{X}}_t \equiv \boldsymbol{X}_t(\hat{\boldsymbol{\beta}})$, and $\hat{z}_t \equiv z_t(\hat{\boldsymbol{\gamma}})$. The DLR (14.45) is actually a simplified version of the DLR that one obtains initially. First, $\hat{\sigma}_1$ times the original regressor for σ_1 has been subtracted from the original regressor for α. Then the regressors corresponding to $\boldsymbol{\beta}$ and σ_1 have been multiplied by $\hat{\sigma}_1$, and the regressor corresponding to α has been multiplied by $\hat{\sigma}_2$. None of these modifications affects the subspace spanned by the columns of the regressor, and hence none of them affects the test statistic(s) one obtains. The last column of the regressor matrix in (14.45) is the one that corresponds to α. The other columns should be orthogonal to the regressand by construction.

Similarly, if we wish to test H_2, we can replace $\boldsymbol{\beta}$ and σ_1 by ML estimates $\hat{\boldsymbol{\beta}}$ and $\hat{\sigma}_1$ so that H_C becomes

$$H_C''\colon\ (1-\alpha)\left(\frac{y_t - x_t(\hat{\boldsymbol{\beta}})}{\hat{\sigma}_1}\right) + \alpha\left(\frac{\log y_t - z_t(\boldsymbol{\gamma})}{\sigma_2}\right) = \varepsilon_t.$$

It is then straightforward to test H_2 against H_C'' by means of the DLR

$$\begin{bmatrix} \dfrac{\log y_t - \hat{z}_t}{\hat{\sigma}_2} \\ 1 \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{Z}}_t & \dfrac{\log y_t - \hat{z}_t}{\hat{\sigma}_2} & \hat{x}_t - y_t \\ \mathbf{0} & -1 & \hat{\sigma}_2 y_t \end{bmatrix} \begin{bmatrix} \boldsymbol{b} \\ s \\ a \end{bmatrix} + \text{residuals}. \tag{14.46}$$

Once again, this is a simplified version of the DLR that one obtains initially, and the last column of the regressor matrix is the one that corresponds to α.

The tests we have just discussed evidently generalize very easily to models involving any sort of transformation of the dependent variable, including Box-Cox models and other models in which the transformation depends on one or more unknown parameters. For more details, see Davidson and MacKinnon (1984a). It should be stressed that the artificial compound model (14.44) is quite arbitrary. Unlike the similar-looking model for regression models that was employed in Section 11.3, it does not yield tests asymptotically equivalent