

the standard normal distribution, this probability is

$$P(\alpha, \lambda) \equiv 1 - \Phi(c_\alpha - \lambda) + \Phi(-c_\alpha - \lambda). \quad (12.36)$$

In order to find the inverse power function corresponding to (12.36), we let  $P(\alpha, \lambda) = \pi$  for some desired level of power  $\pi$ . This equation implicitly defines the inverse power function. It is easy to check from (12.36) that  $P(\alpha, -\lambda) = P(\alpha, \lambda)$ . Thus, if  $P(\alpha, \lambda) = \pi$ , then  $P(\alpha, -\lambda) = \pi$  also. However, the nonuniqueness of  $\lambda$  would not arise if we were to square the test statistic to obtain a  $\chi^2$  form. No closed-form expression exists giving the (absolute) value of  $\lambda$  as a function of  $\alpha$  and  $\pi$  in the present example, but for any given arguments  $\lambda$  is not hard to calculate numerically.

What interpretation should we give to the resulting function  $\lambda(\alpha, \pi)$ ? If we square the asymptotically normal statistic (12.35) in order to obtain a  $\chi^2$  form, the result will have a limiting distribution of  $\chi^2(1, \Lambda)$  with  $\Lambda = \lambda^2$ . Then it appears that  $\Lambda = (\lambda(\alpha, \pi))^2$  is asymptotically the smallest NCP needed in order that a test of size  $\alpha$  based on the square of (12.35) should have probability at least  $\pi$  of rejecting the null.

Let the nonlinear regression model be written, as usual, as

$$\mathbf{y} = \mathbf{x}(\boldsymbol{\beta}) + \mathbf{u}, \quad (12.37)$$

where the parameter of interest  $\theta$  is a component of the parameter vector  $\boldsymbol{\beta}$ . If we denote by  $\mathbf{X}_\theta$  the derivative of the vector  $\mathbf{x}(\boldsymbol{\beta})$  with respect to  $\theta$ , evaluated at the parameters  $\boldsymbol{\beta}_0$ , and by  $\mathbf{M}_X$  the projection off all the columns of  $\mathbf{X}(\boldsymbol{\beta})$  other than  $\mathbf{X}_\theta$ , then the asymptotic variance of the least squares estimator  $\hat{\theta}$  is  $\sigma_0^2(\mathbf{X}_\theta^\top \mathbf{M}_X \mathbf{X}_\theta)^{-1}$ , where  $\sigma_0^2$  is the variance of the components of  $\mathbf{u}$ . If we consider a DGP with a parameter  $\theta \neq \theta_0$ , then for a given sample size  $n$ , the parameter  $\delta$  of the drifting DGP becomes  $n^{1/2}(\theta - \theta_0)$ , and  $\Lambda = \lambda^2$  becomes

$$\Lambda = \frac{1}{\sigma_0^2}(\theta - \theta_0)^2 \mathbf{X}_\theta^\top \mathbf{M}_X \mathbf{X}_\theta. \quad (12.38)$$

This may be compared with the general expression (12.26). Now let  $\theta(\alpha, \pi)$  be the value of  $\theta$  that makes  $\Lambda$  in (12.38) equal to  $(\lambda(\alpha, \pi))^2$  as given above by the inverse power function. We see that, within an asymptotic approximation, DGPs with values of  $\theta$  closer to the  $\theta_0$  of the null hypothesis than  $\theta(\alpha, \pi)$  will have probability less than  $\pi$  of rejecting the null on a test of size  $\alpha$ .

We should be unwilling to regard a failure to reject the null as evidence against some other DGP or set of DGPs if, under the latter, there is not a fair probability of rejecting the null. What do we mean by a “fair probability” here? Some intuition on this matter can be obtained by considering what we would learn in the present context by using a standard tool of conventional statistical inference, namely, a confidence interval. Armed with the estimate  $\hat{\theta}$  and an estimate of its standard error,  $\hat{\sigma}_\theta$ , we can form a confidence interval