We must now show that the SSR from regression (7.50) is asymptotically equal to minus the second term in expression (7.49). This SSR is

$$\big\|\boldsymbol{P}_W\big(\boldsymbol{y}-\boldsymbol{x}(\check{\boldsymbol{\beta}})-\check{\boldsymbol{X}}\check{\boldsymbol{b}}\big)\big\|^2,$$

where $\check{\boldsymbol{b}}$ is the vector of parameter estimates from OLS estimation of (7.50). Recall from the results of Section 6.6 on one-step estimation that $(\tilde{\boldsymbol{\beta}}-\check{\boldsymbol{\beta}})$ is asymptotically equal to the estimate $\check{\boldsymbol{b}}$ from the GNR (7.38). Thus

$$\boldsymbol{P}_W\big(\boldsymbol{y}-\boldsymbol{x}(\check{\boldsymbol{\beta}})-\check{\boldsymbol{X}}\check{\boldsymbol{b}}\big)\overset{a}{=}\boldsymbol{P}_W\boldsymbol{y}-\boldsymbol{P}_W\boldsymbol{x}(\check{\boldsymbol{\beta}})-\boldsymbol{P}_W\check{\boldsymbol{X}}(\tilde{\boldsymbol{\beta}}-\check{\boldsymbol{\beta}}). \tag{7.52}$$

But a first-order Taylor expansion of $\boldsymbol{x}(\tilde{\boldsymbol{\beta}})$ about $\boldsymbol{\beta}=\check{\boldsymbol{\beta}}$ gives

$$\boldsymbol{x}(\tilde{\boldsymbol{\beta}})\cong\boldsymbol{x}(\check{\boldsymbol{\beta}})+\boldsymbol{X}(\check{\boldsymbol{\beta}})(\tilde{\boldsymbol{\beta}}-\check{\boldsymbol{\beta}}).$$

Subtracting the right-hand side of this expression from $\boldsymbol{y}$ and multiplying by $\boldsymbol{P}_W$ yields the right-hand side of (7.52). Thus we see that the SSR from regression (7.50) is asymptotically equal to

$$\big\|\boldsymbol{P}_W\big(\boldsymbol{y}-\boldsymbol{x}(\tilde{\boldsymbol{\beta}})\big)\big\|^2,$$

which is the second term of (7.49). We have therefore proved that the difference between the restricted and unrestricted values of the criterion function, expression (7.49), is asymptotically equivalent to the explained sum of squares from the GNR (7.38). Since the latter can be used to construct a valid test statistic, so can the former.

This result is important. It tells us that we can always construct a test of a hypothesis about $\boldsymbol{\beta}$ by taking the difference between the restricted and unrestricted values of the criterion function for IV estimation and dividing it by anything that estimates σ^2 consistently. Moreover, such a test will be asymptotically equivalent to taking the explained sum of squares from the GNR evaluated at $\check{\boldsymbol{\beta}}$ and treating it in the same way. Either of these tests can be turned into an asymptotic F test by dividing numerator and denominator by their respective degrees of freedom, r and $n-k$. Whether this is actually a good thing to do in finite samples is unclear, however.

7.8 Identification and Overidentifying Restrictions

Identification is a somewhat more complicated matter in models estimated by IV than in models estimated by least squares, because the choice of instruments affects whether the model is identified or not. A model that would not be identified if it were estimated by least squares will also not be identified if it is estimated by IV. However, a model that would be identified if it were estimated by least squares may not be identified if it is estimated by IV using a