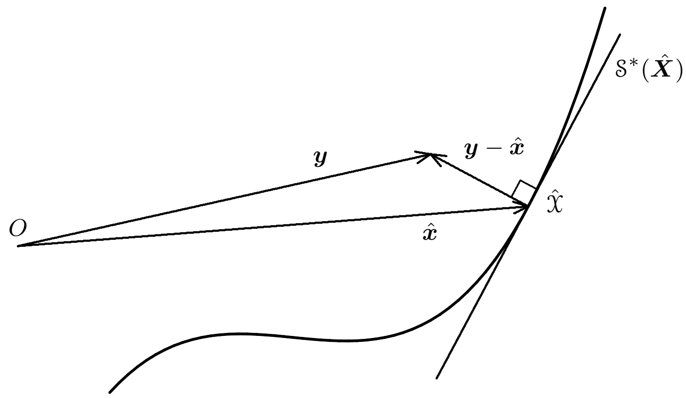


**Figure 2.3** A regressand $\boldsymbol{y}$ projected onto a nonlinear manifold

also with $\mathcal{S}^*(\bar{\boldsymbol{X}})$. However, in this case it is evident that $\hat{\mathcal{X}}$ corresponds to a global minimum, $\mathcal{X}''$ to a local minimum, and $\mathcal{X}'$ to a local maximum of $SSR(\boldsymbol{\beta})$. Thus we see once again that a point which satisfies the first-order conditions does not necessarily yield NLS estimates.

It should be clear from these figures that the amount of nonlinearity in the regression function $\boldsymbol{x}(\boldsymbol{\beta})$ is very important. When $\boldsymbol{x}(\boldsymbol{\beta})$ is almost linear, nonlinear least squares is very similar to ordinary least squares. When $\boldsymbol{x}(\boldsymbol{\beta})$ is very nonlinear, however, all sorts of strange things can happen. Figure 2.4 only hints at these, since there are many different ways for multiple values of $\boldsymbol{\beta}$ to satisfy the first-order conditions (2.05) when $\mathcal{X}$ is a high-dimensional manifold.

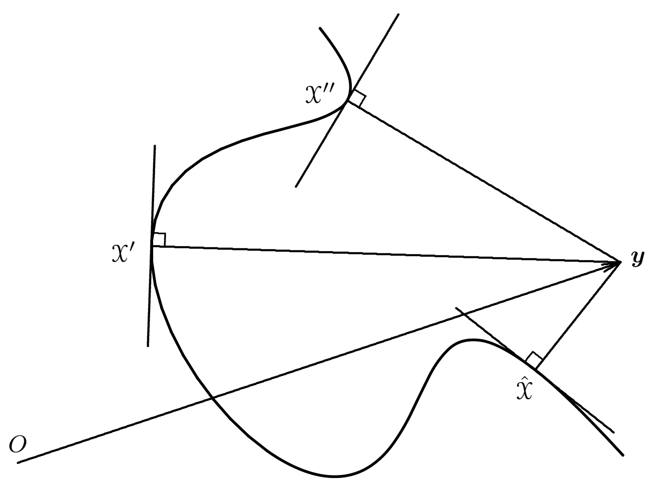


**Figure 2.4** A highly nonlinear manifold