Consequently, the matrix (18.69), evaluated at the ML estimates, becomes

$$-\hat{\boldsymbol{Y}}^{\top}(\boldsymbol{Y}\hat{\boldsymbol{\Gamma}} - \boldsymbol{X}\hat{\boldsymbol{B}})\hat{\boldsymbol{\Sigma}}^{-1}.$$

Now at last we can select the elements of the two partial derivative matrices which are actually zero when evaluated at the ML estimates. The parameters that appear in the i^{th} equation are found in the i^{th} columns of the matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{B}$, and so the appropriate partial derivatives are found in the i^{th} columns of the partial derivative matrices. For the matrix corresponding to $\boldsymbol{B}$, this column is $\boldsymbol{X}^{\top}(\boldsymbol{Y}\hat{\boldsymbol{\Gamma}} - \boldsymbol{X}\hat{\boldsymbol{B}})(\hat{\boldsymbol{\Sigma}}^{-1})_i$. From this column we wish to select only those rows for which the corresponding element of the column $\boldsymbol{B}_i$ is unrestricted, that is, the elements corresponding to the $n \times k_i$ matrix $\boldsymbol{X}_i$. Since in order to select rows of a matrix product, we need only select the corresponding rows of the left-most factor, the zero elements are those of the k_i–vector $\boldsymbol{X}_i^{\top}(\boldsymbol{Y}\hat{\boldsymbol{\Gamma}} - \boldsymbol{X}\hat{\boldsymbol{B}})(\hat{\boldsymbol{\Sigma}}^{-1})_i$.

By exactly similar reasoning, we find that, for each $i = 1, \ldots, g$, the g_i–vector $\hat{\boldsymbol{Y}}_i^{\top}(\boldsymbol{Y}\hat{\boldsymbol{\Gamma}} - \boldsymbol{X}\hat{\boldsymbol{B}})(\hat{\boldsymbol{\Sigma}}^{-1})_i$ is zero, where $\hat{\boldsymbol{Y}}_i$ contains only those columns of $\hat{\boldsymbol{Y}}$ that correspond to the matrix $\boldsymbol{Y}_i$ of endogenous variables included as regressors in the i^{th} equation. If we write $\hat{\boldsymbol{Z}}_i \equiv [\boldsymbol{X}_i \quad \hat{\boldsymbol{Y}}_i]$, then all the first-order conditions corresponding to the parameters of the i^{th} equation can be written as

$$\hat{\boldsymbol{Z}}_i^{\top}\big(\boldsymbol{Y}\hat{\boldsymbol{\Gamma}} - \boldsymbol{X}\hat{\boldsymbol{B}}\big)(\boldsymbol{\Sigma}^{-1})_i = \mathbf{0}.$$

These conditions can be further simplified. Note that

$$\begin{aligned}(\boldsymbol{Y}\hat{\boldsymbol{\Gamma}} - \boldsymbol{X}\hat{\boldsymbol{B}})(\hat{\boldsymbol{\Sigma}}^{-1})_i &= \sum_{j=1}^{g} \hat{\sigma}^{ij}\big(\boldsymbol{Y}\hat{\boldsymbol{\Gamma}}_j - \boldsymbol{X}\hat{\boldsymbol{B}}_j\big) \\ &= \sum_{j=1}^{g} \hat{\sigma}^{ij}\big(\boldsymbol{y}_j - \boldsymbol{Z}_j\hat{\boldsymbol{\delta}}_j\big).\end{aligned}$$

The full set of first-order conditions defining the FIML estimates can thus be written as

$$\sum_{j=1}^{g} \hat{\sigma}^{ij}\hat{\boldsymbol{Z}}_i^{\top}\big(\boldsymbol{y}_j - \boldsymbol{Z}_j\hat{\boldsymbol{\delta}}_j\big) = \mathbf{0}, \quad \text{for } i = 1, \ldots, g. \tag{18.72}$$

The conditions (18.72) are now in a form very similar indeed to that of the conditions (18.63) that define the 3SLS estimator. In fact, if we let $\bar{\boldsymbol{Y}}_i$ denote the $n \times g_i$ matrix of fitted values from the *unrestricted* reduced form, so that $\bar{\boldsymbol{Y}}_i = \boldsymbol{P}_X\boldsymbol{Y}_i$ for $i = 1, \ldots, g$, then

$$\boldsymbol{P}_X\boldsymbol{Z}_i = \boldsymbol{P}_X\big[\,\boldsymbol{X}_i \quad \boldsymbol{Y}_i\,\big] = \big[\,\boldsymbol{X}_i \quad \bar{\boldsymbol{Y}}_i\,\big] \equiv \bar{\boldsymbol{Z}}_i.$$

Thus the conditions (18.63) that define the 3SLS estimator can be written as

$$\sum_{j=1}^{g} \tilde{\sigma}^{ij}\bar{\boldsymbol{Z}}_i^{\top}\big(y_j - \boldsymbol{Z}_j\tilde{\boldsymbol{\delta}}_j\big) = \mathbf{0}. \tag{18.73}$$