they restrict themselves to fully specified parametric models capable of being estimated by maximum likelihood. We will, however, make use of one of their specific examples as a concrete illustration of a number of points.

Let the $1 \times g$ vector $\boldsymbol{Y}_t$ denote the t^{th} observation on a set of variables that we wish to model as a simultaneous process, and let the $1 \times k$ vector $\boldsymbol{X}_t$ be the t^{th} observation on a set of explanatory variables, some or all of which may be lagged $\boldsymbol{Y}_t$'s. We may write an, in general nonlinear, simultaneous equations model as

$$\boldsymbol{h}_t(\boldsymbol{Y}_t, \boldsymbol{X}_t, \boldsymbol{\theta}) = \boldsymbol{U}_t, \tag{18.04}$$

where $\boldsymbol{h}_t$ is a $1 \times g$ vector of functions, somewhat analogous to the regression function of a univariate model, $\boldsymbol{\theta}$ is a p–vector of parameters, and $\boldsymbol{U}_t$ is a $1 \times g$ vector of error terms. The linear model (18.01) is seen to be a special case of (18.04) if we rewrite it as

$$\boldsymbol{Y}_t \boldsymbol{\Gamma} = \boldsymbol{X}_t \boldsymbol{B} + \boldsymbol{U}_t$$

and define $\boldsymbol{\theta}$ so that it consists of all the elements of $\boldsymbol{\Gamma}$ and $\boldsymbol{B}$ which have to be estimated. Here $\boldsymbol{X}_t$ and $\boldsymbol{Y}_t$ are the t^{th} rows of the matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$. A set of (conditional) moment conditions could be based on (18.04), by writing

$$E\big(\boldsymbol{h}_t(\boldsymbol{Y}_t, \boldsymbol{X}_t, \boldsymbol{\theta})\big) = \boldsymbol{0},$$

where the expectation could be interpreted as being conditional on some appropriate information set.

Definition 18.1.

The explanatory variables $\boldsymbol{X}_t$ are **predetermined** in equation i of the model (18.04), for $i = 1, \ldots, g$, if, for all $t = 1, \ldots, n$,

$$\boldsymbol{X}_t \perp\!\!\!\perp u_{i,t+s} \quad \text{for all } s \geq 0.$$

Here the symbol $\perp\!\!\!\perp$ is used to express statistical independence. The definition applies to any context, such as the time-series one, in which there is a natural ordering of the observations. The next concept does not require this.

Definition 18.2.

The explanatory variables $\boldsymbol{X}_t$ are **strictly exogenous** in equation i of (18.04) if, for all $t = 1, \ldots, n$,

$$\boldsymbol{X}_t \perp\!\!\!\perp \boldsymbol{U}_s \quad \text{for all } s = 1, \ldots, n.$$

If (18.04) represents a structural form, then either predeterminedness or strict exogeneity allows us to treat this form as a characterization of the process generating $\boldsymbol{Y}_t$ conditional on $\boldsymbol{X}_t$. Thus we may, for example, write down a loglikelihood function based on (18.04), which can be maximized in