Hence the DLR for the simple Box-Cox model, (14.04) with $\tau(y_t, \lambda)$ given by the Box-Cox transformation, is

$$\begin{bmatrix} \frac{1}{\sigma} u_t(y_t, \boldsymbol{\beta}, \lambda) \\ 1 \end{bmatrix} \tag{14.33}$$

$$= \begin{bmatrix} \frac{1}{\sigma}\boldsymbol{X}_t(\boldsymbol{\beta}) & \dfrac{-(\lambda y_t^\lambda \log y_t - y_t^\lambda + 1)}{\sigma\lambda^2} & \dfrac{u_t(y_t, \boldsymbol{\beta}, \lambda)}{\sigma^2} \\ \mathbf{0} & \log y_t & -\frac{1}{\sigma} \end{bmatrix} \begin{bmatrix} \boldsymbol{b} \\ a \\ s \end{bmatrix} + \text{residuals},$$

where $\boldsymbol{b}$ is a k–vector of coefficients corresponding to $\boldsymbol{\beta}$, a and s are scalar coefficients corresponding to λ and σ, and

$$u_t(y_t, \boldsymbol{\beta}, \lambda) \equiv B(y_t, \lambda) - x_t(\boldsymbol{\beta}).$$

If the DLR (14.33) is evaluated at unrestricted ML estimates $\hat{\boldsymbol{\theta}} \equiv (\hat{\boldsymbol{\beta}}, \hat{\lambda}, \hat{\sigma})$, all the estimated coefficients will be zero. Since the first-order conditions for σ imply that

$$\hat{\sigma} = \left(\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t^2\right)^{1/2},$$

the total sum of squares from the artificial regression will be $2n$. Thus the OLS covariance matrix estimate will simply be $\big(2n/(2n-k-2)\big)(\hat{\boldsymbol{R}}^\top\hat{\boldsymbol{R}})^{-1}$, where $\hat{\boldsymbol{R}}$ denotes the matrix of regressors that appears in (14.33), evaluated at the ML estimates. By the fundamental result (14.20), this OLS covariance matrix provides a valid estimate of the asymptotic covariance matrix of the ML estimator $\hat{\boldsymbol{\theta}}$.

It is clear from (14.33) that this asymptotic covariance matrix is not block-diagonal between $\boldsymbol{\beta}$ and the other parameters. Forming the matrix $\boldsymbol{R}^\top\boldsymbol{R}$, dividing by n, and taking probability limits, we see that the $(\boldsymbol{\beta}, \boldsymbol{\beta})$ block of the information matrix $\mathfrak{I}(\boldsymbol{\theta})$ is simply

$$\sigma^{-2} \operatorname*{plim}_{n\to\infty}\Big(\frac{1}{n}\boldsymbol{X}^\top(\boldsymbol{\beta})\boldsymbol{X}(\boldsymbol{\beta})\Big), \tag{14.34}$$

as it would be if this were a nonlinear regression model. The (σ, σ) element is simply $2/\sigma^2$, which again is what it would be if this were a nonlinear regression model. But $\mathfrak{I}(\boldsymbol{\theta})$ also contains a (λ, λ) element, a (λ, σ) element, and a $(\boldsymbol{\beta}, \lambda)$ row and column, all of which are clearly nonzero. For example, the element corresponding to β_i and λ is

$$-\operatorname*{plim}_{n\to\infty}\left(\frac{1}{n\sigma^2\lambda^2}\sum_{t=1}^{n} X_{ti}(\boldsymbol{\beta})\big(\lambda y_t^\lambda \log y_t - y_t^\lambda + 1\big)\right).$$

The (λ, λ) and (λ, σ) elements can also be obtained in a straightforward fashion and are easily seen to be nonzero.