Nosiness, Loss Aversion and the Stability of Sen's Paradox[1]

Chetan Dave[2]
The University of Texas at Dallas

Stefan Dodds[3]
Carleton University

November 2, 2008

[1]We would like to thank Sheryl Ball, Christopher Barrington-Leigh, Nathan Berg, Rachel Croson, Frances Woolley and participants at the 2008 Canadian Economic Association meetings and the University of Pennsylvania for valuable comments. All errors are those of the authors.

[2]Address: University of Texas at Dallas; School of Economic, Political and Policy Sciences; 800 W. Campbell Rd.; Richardson, TX 75080. Phone: (972) 883-2306. Fax: (972) 883-6486. Email: cdave@utdallas.edu.

[3]Address: Carleton University; Department of Economics; 1125 Colonel By Drive, Ottawa, ON, Canada, K1S 5B6. Phone: (613) 520-2600 xt. 3773. Email: sdodds@ccs.carleton.ca.

Abstract

Sen's liberal paradox showed that collective choices cannot simultaneously satisfy Paretian and liberal ethics when individuals have 'nosy' preferences. Our paper uses behavioral game theory to study this problem. First, we show how the paradox can be recast as an extended game in which individuals value contractions in their set of strategies more than equivalent gains in strategies, and have preferences over the strategies employed by their opponents. Second, we present a natural interpretation of paradox resolution in this context: namely, the situation where individuals achieve Pareto efficient outcomes by agreeing to restrict or expand the strategy sets from which they can choose. Third, we show how these resolutions jointly depend on individuals' nosiness and loss-aversion. Our results suggest that 'pragmatic' resolutions of Sen's paradox can exist, a conjecture that lends itself to experimental investigation.

1 Introduction

The Pareto Principle declares that if everyone in a society prefers some social state of the world (x) to another state (y) then the aggregated social ranking should reflect this as well. 'Minimal' Liberalism requires that no individual's preference relation between social states, say z and w, be violated in the social choice, provided that the position of other individuals is unaffected across states z and w. Both of these principles can be appealing to economists as they evaluate the normative features of various policies.

However, Sen's (1970) liberal paradox demonstrates that the Pareto Principle and Minimal Liberalism are mutually inconsistent. The paradox shows that if individuals possess nosy preferences — that is, they form rankings over what others should do even though their own material well-being is unaffected by others' actions — then these two axioms may not jointly be satisfied in a social choice. As an elegant proof by contradiction, the paradox adds to the list of social choice impossibilities. In this paper we revisit this impossibility result using the tools of behavioral game theory to gain insight into possible 'pragmatic' resolutions to the paradox.

The canonical example of the Paradox concerns the censorship of otherwise private activities. As long as individuals care sufficiently strongly about what others do in their own private lives, there can be no accepted social state of the world that respects all individual preference orderings as well as satisfies the Pareto Principle. In such settings, universal censorship can lead to Pareto efficiency since all parties modify their personal behavior to satisfy others' nosiness. But imposing such an outcome necessarily violates the liberty of censored parties.

An especially straightforward interpretation of the paradox — which we take as the point of departure for our analysis — is due to Aldrich (1977). He argues that the paradox can be interpreted game-theoretically as a two-person Prisoner's Dilemma. In this framework there are two plausible social choices when agents make strategic decisions: the Cooperative outcome and the Nash equilibrium. The former satisfies the Pareto Principle but not Minimal Liberalism, while the latter satisfies Minimal Liberalism but not the Pareto Principle. Ultimately, Aldrich (1977) is not prescriptive about which state (Cooperative or Nash) should occur. Instead, he highlights the fact that the paradoxical social situation outlined by Sen (1970) is amenable to a game theoretic interpretation.

The paradox-as-game interpretation is at odds with that of Sen (1976), who argues that equilibria in such games are about the mutual anticipation of actions as opposed to the exercising of inherent rights. In Sen's (1970) view, rights operate over final outcomes, not strategies. Seidl (1996) refers to Sen's conception of rights as consequentialist, as opposed to the game form approach which is deontological.[1] In the game form approach, there is no normative weight on outcomes, only on the 'rules of the game' (i.e. the set of permissible strategies).

For the sake of brevity, we omit a detailed comparison of these approaches to social choice theory and instead focus on the game theoretic approach, which interprets rights as strategies. We re-examine a classic question: namely, can society overcome the paradox? Our concern is with pragmatic, as opposed to axiomatic, resolutions to the paradox. In the present context, this means identifying situations where individuals will agree to sets of rights ex ante which lead to Pareto-efficient outcomes ex post. This goal may also be phrased negatively: when will individuals fail to agree to distributions of rights that in turn produce efficient outcomes?

To investigate this question, we consider a simple two-person social environment. When interacting with one another, each individual enjoys a set of rights which can be exercised or not. Following the game-form interpretation, these rights are the strategies each player is permitted to

[1] In moral philosophy, deontology is the view that morality either forbids or permits actions. For example, a deontological moral theory might hold that lying is wrong, even if it produces good consequences.

employ. Rights exercised (strategies chosen) have no direct impact on the payoff of the other individual. However, individuals have nosy preferences, in that they care about the rights exercised by others in addition to those exercised by themselves. Preferences are a weighted sum of personal and others' payoffs, but the weights assigned to others' payoffs can change depending on the particular right they choose to exercise. Given a distribution of rights (a potentially non-symmetric set of strategies assigned to each player), predicted outcomes are simply Nash equilibria.

Our objective is to find distributions of rights which (a) generate Pareto-efficient Nash equilibria and (b) are stable. Stability means that both individuals prefer the existing distribution of rights to a perturbation which creates new rights or eliminates existing rights with some probability. We interpret a stable distribution of rights as one which would be supported constitutionally (i.e. a 'bill of rights') against these random perturbations. Sen's Paradox is then recast as a conflict between conditions (a) and (b) above: we seek to find conditions under which stable distributions of rights fail to produce Pareto-efficient outcomes. In other words, when might we expect 'inefficient' constitutions to be enshrined, and the tension between efficiency and liberty to be preserved?

In this paper we argue that by adopting Segal and Sobel's (2007) representation theorem of preferences defined over own and others strategies, we can enrich the game theoretic interpretation of Aldrich (1977) by explicitly modeling individual preferences as two-dimensional (intrinsic and 'nosy'). Application of the theorem to the environment suggested by Aldrich (1977) permits us to characterize equilibria as functions of nosiness. We first demonstrate with a fixed distribution of rights how the degree of nosiness determines the efficiency of outcomes. Nosiness is introduced intuitively by using strategy-dependent social preferences. We then turn our attention to examining the stability of rights-distributions. We show that by allowing individuals to be loss averse over strategies — in the sense that a loss of an element of the strategy space is valued differently than a gain — we can characterize a much richer and realistic set of behavior. We investigate the possibility of 'overcoming' the paradox, by identifying those situations in which agents would agree to constitutionalize rights-distributions which in turn produce efficient outcomes. Next, we show how the paradox can be sustained if agents exhibit loss aversion over own-strategies, and how it disappears when losses and gains of rights are treated symmetrically. In this way, loss-aversion over rights is shown to be necessary, but not sufficient, for the paradox to persist. Segal and Sobel (2007) type preferences are of special importance, since the feasibility of overcoming the paradox depends on the degree of nosiness that they explicitly incorporate. This is absent in the standard view of the paradox as a simple prisoners' dilemma.

2 Sen's Paradox with Social Preferences

Imagine a particularly simple social situation. There are two individuals named L and P, who can in theory each take some action (denoted '1') or not (denoted '0'). Combinations of action and inaction generate social states, of which there are four possibilities: $x = \{0, 0\}$, $y = \{1, 0\}$, $z = \{0, 1\}$ and $w = \{1, 1\}$. For example, $y = \{1, 0\}$ indicates that individual L takes the action, while individual P does not.

2.1 Basic Structure of the Paradox

The key ingredient of Sen's Paradox is the 'nosiness' of individual preferences. Sen's original analysis assumes that individuals have complete and consistent preferences over social states. Since these social states incorporate the 'features' of each individual (i.e. whether or not each takes the action), nosiness is captured by preference orderings which appear to give place greater importance on the features of other individuals relative to one's own. One particular such set of individual preference profiles is given by:

$$w \succ z \succ y \succ x \qquad \text{For } L \tag{1}$$

$$x \succ z \succ y \succ w \qquad \text{For } P \tag{2}$$

Given these individual orderings, an axiomatic analysis of the possible social outcomes can be considered. Clearly, z is Pareto-preferred to y. Next, Sen's (1970) principle of Minimal Liberalism (ML) stipulates that the social ordering of states should reflect individual orderings, whenever an individual is *decisive* over a set of alternatives. An individual is called decisive between states when only his personal features change between states, whereas the features of other individuals are fixed between these same states. Using ML, individual P is decisive over pairs $< w, y >$ and $< x, z >$ and individual L is decisive over pairs $< x, y >$ and $< w, z >$. This axiom yields the social ranking $y \succ x \succ z$ (or equivalently, $y > w > z$) since L prefers y to x and P prefers x to z. Transitivity implies the social ranking $y \succ z$, by minimal liberalism. However, z is Pareto-preferred to y. Thus any choice rule which selects state y must violate the Pareto principle, but any choice rule that selects z must violate minimal liberalism.[2]

One can apply Sen's classic example of individuals reading a contentious book (action 1) or not (action 0) to the above situation, where individual L is labeled Lewd and individual P is labeled Prude (as inspection of their preference profiles indicates). Lewd personally prefers reading to not regardless of what Prude does, while Prude personally prefers not to read regardless of Lewd's action. But each is also nosy: independent of his own actions, Lewd prefers that Prude read, while independent of her own actions, Prude prefers that Lewd not read. The Pareto efficient outcome

[2] It is easy to see that x and w are also Pareto efficient outcomes, since neither is Pareto-dominated. However, if either becomes the social choice, ML must be violated. To see why, suppose x is the social choice. Since L is decisive over $< x, y >$, invoking ML requires that L be able to select state y for society. Similarly, making w the social choice violates the decisiveness of P over $< w, y >$.

— for only Prude to read — embodies a form of censorship where each is compelled to take an action which he or she does not intrinsically prefer.

2.2 Interpretation of the Paradox as a Game

The consequentialist view of the paradox is agnostic about the processes which generate social outcomes. But it is straightforward to reformulate the above scenario as a game where (i) individuals each begin with the right to read the book or not, and (ii) outcomes are generated by the free exercise of these rights. This is the insight of Aldrich (1977). In his view, the ML criterion operates within a particular game since individuals can rule out certain outcomes (i.e. be decisive over certain outcomes) by making particular strategy choices.

Continuing with the classic book example, let R indicate that an individual chooses to read the book, and D indicate that an individual does not so choose. The game has the general representation:

		Prude	
		R	D
Lewd	R	w	y
	D	z	x

Aldrich (1977) contends that *when* payoff structures are *such that* y is chosen as a unique Nash Equilibrium, the liberal paradox is illustrated, since the free exercise of rights (to read the book or not) leads to inefficiency. The Prisoner's dilemma is an example of such a game.

3 Characterizing The Paradox with Social Preferences

There are two key limitations to the simple paradox-as-game interpretation outlined above. First, the concept of 'nosiness' — key to Sen's initial formulation — is eliminated, or implicitly embedded in the material payoffs of players in the game. To maintain Sen's original conception of nosiness, individual preferences need to operate directly over the *actions* employed by other individuals. Second, very limited scope is given to the concept of liberalism, which is taken in the paradox-as-game interpretation to be simply a free choice from the pre-determined set of possible actions. However, an early axiomatic 'resolution' of the paradox due to Gibbard (1974), involves allowing individuals to reach Pareto-efficient outcomes by waiving certain rights. By making the set of rights negotiable, individuals may at least in theory be willing to concede freedom of action in return for Pareto-preferred outcomes. In a game-theoretic setting, this process involves individuals agreeing to the rules of the game before the game is played, i.e. they must decide first what actions will be allowed and who will be allowed to choose them. Endogenizing the strategy sets of players gives them the 'right to choose their rights,' and is therefore a broader conception of liberalism than that implied by a fixed-strategy environment.[3]

Our method of characterizing the existence of the paradox is therefore a two-stage process. We first expand the game with fixed strategy sets to explicitly include social preferences over others' actions. By doing so, we demonstrate cases where free choices generate inefficient outcomes. Next, given these preferences, we investigate in what situations individuals will agree (or not) to particular distributions of permissible strategies ex ante. We characterize the paradox as the situation where individuals prefer distributions which generate inefficient equilibria ex post. Again, we recall that

[3]This is known as the game form approach to rights: see e.g. Gaertner, Pattenaik and Suzumura (1992) and Pattenaik (1994). The game-form approach endogenizes individuals' strategy sets by modeling the choice of strategies which are permitted in a game.

our objective is to characterize the paradox as a 'behavioral' context-dependant game which allows us to investigate the possibility of pragmatic resolutions.

3.1 Fixed Strategy Sets and Social Preferences

We return to the two-person book-reading case to motivate our argument. Both individuals begin with the largest scope of action: to either read the book or not. In Sen's original conception, Lewd personally prefers to read a particular book and Prude personally prefers not to read. Let β be the intrinsic payoff to each individual when they choose their preferred action. Any player choosing their less-preferred action ('read' for Prude, 'don't read' for Lewd) receives an intrinsic payoff $\delta < \beta$. Absent social preferences, the book-choice game becomes:

		Prude	
		R	D
Lewd	R	(β, δ)	(β, β)
	D	(δ, δ)	(δ, β)

The Nash equilibrium in this game would trivially be $\{R, D\}$: Lewd reads, Prude does not. It would also be trivially Pareto efficient. Intuitively, one can make the following statement: if players have no concern with their opponent's behavior, then there is no conflict between freedom of action and efficiency.

To introduce social preferences we employ a general characterization suggested by Segal and Sobel (2007). These 'SS-preferences' are particularly appealing since they incorporate not only the possibility that individuals concern themselves either positively or negatively with others' utilities, but also that this concern may be depend on actions chosen by opponents. These preferences take the following form:

$$u_i(\sigma_i, \sigma_j) + a^j_{i,\sigma} u_j(\sigma_i, \sigma_j) \qquad i, j = P, L$$

where $\sigma = (\sigma_i, \sigma_j)$ is a strategy profile, $u_i(\cdot)$ (respectively $u_j(\cdot)$) is player i's (resp. j's) intrinsic utility given a particular outcome, and $a^j_{i,\sigma}$ is the strategy-specific weight player i assigns to j's intrinsic payoff when play is (σ_i, σ_j). The generality of the SS-preferences allows us to capture other-regarding preferences depending on the pattern of behavior, as formalized by $a^j_{i,\sigma}$. Moreover, we can replicate the paradox-as-game interpretation without interpreting payoffs as exclusively pecuniary.

As Segal and Sobel (2007) note, individuals may be willing to sacrifice their intrinsic payoff to see the payoff of their opponent either rise ($a > 0$) or fall ($a < 0$) depending on what strategies they choose to play. These action-dependent interdependencies correspond nicely with the intuition of nosiness: Lewd disapproves of Prude's behavior when Prude decides not to read the book, but approves when Prude decides to read. Thus let $a^P_{L,(\cdot,R)} > 0$ and $a^P_{L,(\cdot,D)} < 0$. Similarly, Prude approves of Lewd's decision to forego reading ($a^L_{P,(D,\cdot)} > 0$), and disapproves of his decision to read ($a^L_{P,(R,\cdot)} < 0$). For simplicity, we let $a > 0$ be the weight placed on approval for either individual, and $-a$ be the weight placed on disapproval.

Grafting these preferences onto the game above, we now have the game:

		Prude	
		R	D
Lewd	R	$(\beta + a\delta,\ \delta - a\beta)$	$(\beta - a\beta,\ \beta - a\beta)$
	D	$(\delta + a\delta,\ \delta + a\delta)$	$(\delta - a\beta,\ \beta + a\delta)$

Note that the choice of both R (for Lewd) and D (for Prude) continue to be dominant regardless of a. But now the social nature of preferences becomes explicit. A higher level of a represents a greater degree of nosiness; if $a = 0$, the game reverts to the simplest case of non-social preferences. But in every case, the 'free' choices of players produces the dominant-strategy equilibrium $\{R, D\}$. The game can therefore resemble the Prisoner's Dilemma, but whether it does so depends on a. Specifically, the unique Nash equilibrium is Pareto efficient if and only if $\beta(1-a) > \delta(1+a)$, or

$$a < \frac{\beta - \delta}{\beta + \delta} \equiv a_e$$

i.e. so long as players are not sufficiently concerned about their partner's behavior.

Remark 1. Given fixed strategy sets with elements R and D for each player, the equilibrium of the book choice game is inefficient if players are sufficiently nosy (a sufficiently high).

Example 1. Let $\beta = 10$ and $\delta = 5$. The game including both intrinsic and other-regarding payoffs is as follows:

		Prude	
		R	D
Lewd	R	(10+5a, 5-10a)	(10-10a, 10-10a)
	D	(5+5a, 5+5a)	(5-10a, 10+5a)

Then $a_e = 1/3$: if each player places a positive or negative weight of at least one third as much as the weight they place on their own payoff, they would each fare better by jointly choosing the option which provides the lowest intrinsic utility. As a changes, so does the game: for example, with $a = 1/10$, we would have:

		Prude	
		R	D
Lewd	R	(10.5, 4.5)	(9.5, 9.5)
	D	(5.5, 5.5)	(4.5, 10.5)

and the dominant-strategy equilibrium $\{R, D\}$ is also Pareto-efficient.

3.2 Variable Action Sets and Loss Aversion

One 'solution' to the paradox involves restricting the set of actions from which individuals can choose. A crucial libertarian requirement is that all involved voluntarily agree to restrictions or expansions on their action sets (strategies). For example, suppose players were required to choose between the following options: (i) Lewd restricts his permissible strategies to the singleton $\{D\}$ and Prude limits hers to $\{R\}$; and (ii) Each remains able to freely choose from the set $\{R, D\}$. Choice (i) is akin to each agent signing a binding collusive agreement, with obvious benefits for each when a is sufficiently large. Choice (ii) is the freedom of choice case which, for a large, leads to a Pareto inefficient outcome. Returning to Example 1, it is reasonable to expect individuals to agree to choice (i) for $a = 1/2$, and likewise to settle on choice (ii) for $a = 1/10$. Such a choice can be characterized as one of constitutional choice: setting the rules of the game before it is actually played. Only if $a = 1/3$ would both individuals be indifferent to either option, since then the extra value of choosing one's preferred private action (R for Lewd, D for Prude) exactly offsets the non-intrinsic (or nosy) loss incurred when the other agent freely makes his or her own preferred private choice.

The constitutional choice made in the above example was not framed in reference to any pre-existing distribution of rights. Our objective is to include this feature and illustrate its relevance to pragmatic resolutions of the paradox. Suppose therefore that both individuals were given the action-set choice problem in a society where rights were already restricted as under option (i). If a were low enough (such that each would benefit from free choice ex post), they ought to prefer the alternate rights distribution where both individuals can choose freely. If the problem were reversed, and individuals started from an initial position where both choices (R and D) were permitted for each agent, they ought to preserve free choice. Adding a constitutional stage — where the set of rights is negotiable — can in principal allows individuals to overcome the ex post inefficiencies by changing the rules of the game ex ante. Individuals could pre-select distributions of rights leading to efficient outcomes, and do so voluntarily.

The key issue to examine is under what conditions individuals might *fail* to agree to limit or expand their strategy sets, even though doing so would result in an ex-post Pareto-efficient outcome. Our conjecture is that loss aversion (over strategies) might act as such a barrier.[4] In other words, loss aversion can limit the effectiveness of the constitutional stage as a device for reaching Pareto-efficient outcomes.

The following two sub-sections illustrate this line of argument. In each section we consider two 'default' distributions of rights ex ante: either (i) the restricted distribution: Lewd must choose $\{D\}$ and Prude must choose $\{R\}$ or (ii) the free-choice distribution: each individual can choose from $\{R, D\}$. We then model individuals as being loss-averse over the set of rights they enjoy, and investigate whether they would prefer either the default distribution or an unstable distribution of rights (where there is uncertainty in the set of rights they and their partners will be permitted ex post).

[4]Loss aversion as a decision making principle is often the basis for explaining the effects of valence framing. Levin et al. (2004) describe three main sources of such frames: risky choice frames, goal based frames and attribute based frames. Whereas economists typically focus on risky choice frames, in this paper we implicitly focus on goal based frames.

By so doing, we examine conditions under which default (stable) distributions of rights are preferred. We argue that Sen's Paradox exists in two cases: first, when the default free-choice distribution is preferred and resulting equilibria are inefficient, and second when the default restricted choice distribution is not preferred, even when doing so would lead to efficient outcomes expost. We can then characterize the existence of Sen's Paradox as a function of (i) the degree of loss aversion over rights and (ii) the degree of nosiness among individuals, and hence give a behavioral game theoretic interpretation of the paradox.

3.3 Case 1: Destabilizing existing rights-distributions

Two default rights distributions are considered: restricted or free-choice. Starting from each distribution, we consider a potential destabilized distribution of rights. Each destabilized distribution represents a risk that the existing rights distribution will change. We treat such changes as exogenous, and take an agnostic view as to their political interpretation. The description of default and destabilized distributions is as follows.

- Free-choice Distribution: The pre-existing condition is for both players to have both rights. With probability p each player retains both rights. With probability $(1-p)$, an individual loses the right to choose his or her preferred action. In other words, Lewd loses the right to choose R with probability $1-p$, and Prude the right to choose D with the same probability. Ex ante, each agent places a weight $-L$ on the potential loss of their preferred right.

- Restricted-choice Distribution: The pre-existing condition is for Lewd being forced to choose D and for Prude being forced to choose R. With probability $1-p$, a player gains the right to choose his or her preferred action; with probability p no such gain occurs. Ex ante, each agent places a weight G on the potential gain of their preferred right.

Given the form of destabilizations above, each ex post distribution of rights occurs with the same probability regardless of the default rights distribution. Moreover, since both players are identically nosy by construction, we need only consider the preferences of a particular individual over rights distributions. The requirement $G < L$ is necessary to impose loss aversion over rights.

Suppose first that free-choice is the default distribution. The dominant-strategy equilibrium (where each player chooses what they individually prefer) results in the payoff of $\beta - a\beta$ for each. Under the de-stabilized rights scenario, each player's expected payoff is:[5]

$$\Pi_f = p^2(\delta + a\delta - L) + p(1-p)(\beta + a\delta) + p(1-p)(\delta - a\beta - L) + (1-p)^2(\beta - a\beta)$$

Simplifying:

$$\Pi_f = (1-p)(\beta - a\beta) + p(\delta(1+a) - L).$$

A player will prefer the de-stabilized situation to the existing guarantee of both rights if $\Pi_f - (\beta - a\beta) > 0$, or:

$$a > \frac{\beta + L - \delta}{\delta + \beta} \equiv \overline{a}.$$

Since $\partial\overline{a}/\partial L > 0$, as the expected loss from losing rights (L) increases, individuals will prefer de-stabilization only if they are increasingly nosy.

Now consider the restricted-choice case as the initial distribution. The unique outcome with restricted rights must be $\{D, R\}$, and the payoff in this case is $\delta + a\delta$. Under the de-stabilized

[5]To compute expected payoffs, we look for Nash equilibria all possible-rights distributions and weighted by the probability of their occurrence. Calculating expected payoffs is straightforward since all possible games are solvable by dominant-strategies.

rights scenario, each player's expected payoff is:

$$\Pi_r = p^2(\delta + a\delta) + p(1-p)(\beta + a\delta + G) + p(1-p)(\delta - a\beta) + (1-p)^2(\beta - a\beta + G).$$

Simplifying, we have:

$$\Pi_r = (1-p)(\beta - a\beta + G) + p(\delta(1+a)).$$

A player will prefer the destabilized situation to the existing restriction of rights if $\Pi_r - (\delta + a\delta) > 0$, or:

$$a < \frac{\beta + G - \delta}{\delta + \beta} \equiv \underline{a}.$$

Again, there is a straightforward interpretation: when individuals are not particularly nosy, they will always be willing to gain rights with some probability.

One can now analyze the stability of the initial rights distributions in the following sense: given a pre-existing distribution of rights, would players be willing to accept a de-stabilization of this distribution where they may gain (or lose) rights? If so, the default distribution cannot be stable, and we would not expect players to agree on the initial distribution as a constitutional guarantee. Recall that an efficient constitutional choice is for players to guarantee restricted rights when $a \geq a_e$.

Remark 2: So long as $a < a_e$ or $a > \overline{a}$, players will never agree to a distribution of rights that produces inefficiency, regardless of the initial distribution of rights.

Suppose for example that a is close to zero. Starting from restricted rights, players will agree to destabilize the distribution in the direction of gains in rights since $a < a_e < \underline{a}$. Starting from free choice, players will agree to preserve rights since $a < a_e < \overline{a}$. Thus, regardless of the initial distribution of rights, no agreement to restrict rights is expected when individuals are relatively nonnosy. When $a < a_e$, this is exactly the desired outcome in a normative sense. Similarly, for $a > \overline{a}$, players should preserve restrictions starting from restrictions (since $\overline{a} > \underline{a}$) and prefer restrictions starting from freedom of choice. Since $a > a_e$, it is normatively undesirable to preserve rights. Thus, for a sufficiently low or sufficiently high, the paradox is not observed: players will tend to choose the 'correct' constitutional distribution of rights regardless of the initial rights-distribution. However, this is not the case for intermediate levels of nosiness.

Remark 3: If $0 < G < L$, then $a_e < \underline{a} < \overline{a}$, and constitutional choices are not efficient for all initial rights-distributions.

Figure 1 depicts this situation for $0 < G < L$. The existence of the paradox can be shown to depend on the initial distribution of rights. Suppose first that $\underline{a} < a < \overline{a}$. These values correspond to region P_2 of the diagram. As described above, preserving freedom of choice in this case is inefficient ex post since a is too high. A restricted rights distribution is preferred if this is the pre-existing distribution, in which case a constitutional choice to preserve restrictions is the correct one. But if freedom of choice is the pre-existing distribution, loss aversion over rights makes players resist restricting rights. Thus, freedom of choice is also (wrongly) stable.

If on the other hand $a_e < a < \underline{a}$, an initial restricted distribution becomes unstable: starting from restricted rights players agree to gain rights, even though both players are sufficiently nosy to prefer restrictions. These values correspond to region P_1 of the diagram. In summary, so long as a inhabits a medium range, we tend to observe the paradox: players may fail to agree to change the pre-existing distribution of rights to produce efficient outcomes.

Two other results are immediately apparent:

Remark 4: If $0 = G = L$, the paradox disappears.

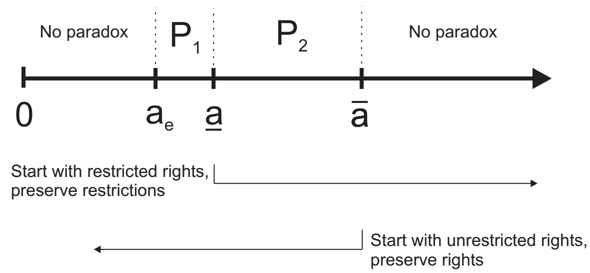

Figure 1:

Remark 5: If $0 < G = L$, region P_2 disappears. Players tend to preserve rights when rights are pre-existing, and fail to preserve restrictions when restrictions are pre-existing. If $0 = G < L$, region P_1 disappears.

A standard resolution of the paradox is to waive rights when their exercise leads to inefficiency. The inability to do so (preservation of the paradox) is captured by region P_2 in the figure. So long as individuals are loss averse over rights (i.e. $G < L$), such a region exists. We have identified an additional incidence of the paradox: namely that individuals may incorrectly fail to preserve restrictions on rights. This outcome is only possible if $G > 0$. This result highlights the sensitivity of the paradox to the initial situation: i.e. the rights that are initially in place when individuals contemplate a constitutional choice.

3.4 Case 2: A 'veil of ignorance' rights distribution

In the previous analysis, individuals formed preferences over preserving or destabilizing rights distributions knowing that they would be in a position to (a) either gain rights (with a restricted initial distribution) or (b) lose rights (starting from free choice). We now consider the question of preserving or restricting rights when the destabilized rights distribution is the same regardless of the default distribution. This requires adding a level of abstraction to the problem, by allowing individuals to form preferences over maintaining an existing rights distribution, or moving to an alternative (unknown) distribution of rights which can itself change.

As before, players start with a default distribution of rights that is either free choice, or restricted. Each player forms preferences either (i) remaining with this distribution or (ii) switching to an uncertain initial (or 'preliminary') distribution which is itself subject to perturbations. Since individuals are uncertain about what their initial position will be under the destabilized distribution, we refer to this scenario as choosing 'behind a veil of ignorance.' Individuals can prefer the existing stable distribution, or choose to swap it for another preliminary distribution which is then perturbed. Under scenario (ii):

- With probability p, a player will be endowed with both rights (i.e. they will be able to choose either R or D)
- With probability $(1-p)/2$ a player will be endowed with a single right: either only R, or only D.

This specifies the preliminary distributions which can occur in the destabilized case.[6] However, the uncertain initial distribution can itself change. Before any actions are chosen by the players, the initial distribution of rights is redrawn by the same random process. For simplicity, we assume that the two draws of rights are independent. If a player's strategy set changes, they will be subject to either a gain or loss in rights. As an example, suppose:

- Prude draws $\{R, D\}$; Lewd draws $\{D\}$. This is the preliminary distribution, which occurs ex ante with probability $p(1-p)/2$.
- Rights are re-drawn. Prude again receives $\{R, D\}$, while Lewd draws $\{R, D\}$ as well. Lewd has gained D relative to the default. There is no gain or loss for Prude.

[6] Note that the game is symmetric, so it doesn't matter that players know whether they are Lewd or Prude *a priori*.

As in the previous section, we assume that loss aversion only applies to preferred strategies. If a player starts without his or her preferred strategy under the preliminary distribution, he places a weight $G \geq 0$ on the possibility of gaining this right. If a player starts with their preferred strategy under the preliminary distribution, he weighs the possibility of losing this right at $-L$. The condition $G < L$ ensures that a player perceives a potential loss of an established right more strongly than the perceived gain of this same right. Regardless of the distribution of rights, it will always be a dominant strategy to choose the preferred action when this right is in a player's strategy set.[7] This allows us to compute the expected payoff of the destabilized scenario to compare with either (i) the restricted rights case or (ii) the freedom of choice case.

With a simple application of Bayes' Rule, we have the following observation:

Remark 6: In the veil of ignorance rights-distribution:

- If a player is ultimately endowed with the preferred right in his strategy set, the probability that this right *will have been gained* is $(1-p)/2$.
- If a player is ultimately not endowed with the preferred right in his strategy set, the probability that the preferred right *will have been lost* is $(1+p)/2$.

Note that as $p \to 1$, we approach the case where both players will be initially endowed with both rights, and can expect to keep both rights when they are redrawn. Thus, gains in rights are unlikely, unconditionally. However, if a player ends up not being endowed with their preferred right when actions are chosen, it is rather likely that this right will have been lost from the initial distribution.

[7] For example, if Lewd can choose between R and D, and Prude must choose D, the social outcome will be $\{R, D\}$: Lewd reads, Prude does not.

Omitting some algebra, the expected payoff to an individual from the veil of ignorance distribution can be calculated as:

$$\Pi = \beta(1-a)\left(\frac{1+p}{2}\right) + \delta(1+a)\left(\frac{1-p}{2}\right) + (1/4)(G-L)(1-p^2).$$

It remains to compare this payoff with that of (i) the restricted rights distribution and (ii) the free-choice distribution. A player prefers to destabilize restricted rights if $\Pi - \delta(1+a) > 0$, or:

$$\left(\frac{1+p}{2}\right)[\beta(1-a) - \delta(1+a)] + (1/4)(G-L)(1-p^2) > 0. \tag{3}$$

A player prefers to destabilize free choice if $\Pi - \beta(1-a) > 0$, or:

$$\left(\frac{1-p}{2}\right)[\delta(1+a) - \beta(1-a)] + (1/4)(G-L)(1-p^2) > 0. \tag{4}$$

Without loss aversion (either $G = L$, or $G = L = 0$), these conditions are identical to the previous efficiency requirement that we should not restrict rights if $\beta(1-a) > \delta(1+a)$ and not allow free choice if $\delta(1+a) > \beta(1-a)$. If so, individuals' preferences between remaining with existing rights distributions versus switching to a destabilized distributions are precisely in line with the normative outcomes. With loss aversion, however, individuals may not preserve existing distributions (free choice or restricted) even though they ought to.

Example 2. Let $\beta = 10$, $\delta = 5$, $G = 0$ and $L = 3$. As before, $a_e = 1/3$. An individual prefers to destabilize the free-choice distribution (inequality (2)) if

$$\frac{5(1-p)(3a-1)}{2} - \frac{3(1-p^2)}{4} > 0. \tag{5}$$

Note that for some $a > 1/3$, this expression will not hold for a range of p. For example, suppose $a = 1/2$, in which case preserving free choice is inefficient. Players will choose to preserve free choice if $3p^2 - 5p + 2 < 0$; i.e. for any $p \in (2/3, 1)$. Similarly, an individual prefers to destabilize the restricted-choice distribution (inequality (1)) if

$$\frac{5(1+p)(1-3a)}{2} - \frac{3(1-p^2)}{4} > 0. \tag{6}$$

For some $a < 1/3$, this expression will not hold for a range of p. For example, suppose $a = 1/4$, in which case we should not restrict choice. Players will nevertheless choose to restrict choice if $6p^2 + 5p - 1 < 0$; i.e. for any $p \in (0, 0.145)$.

A graphical representation of the example is given in figure 2 in the (a, p) parameter space. Expression (4) is shown by the lowest line in the diagram, expression (3) by the highest line, and $a = 1/3$ by the horizontal line. Absent loss aversion, parameter p is irrelevant: so long as $a > 1/3$, players would agree to restrict rights, and if $a < 1/3$, players would agree to preserve rights. Thus regions 1 and 2 in the figure normatively require that restricted rights be preferred, and regions 3 and 4 normatively require that free-choice be preferred. We have the following observations:

Remark 7 Suppose $0 \leq G < L$. Then,

- In region 1, there is no paradox: for a sufficiently high, players will prefer to preserve restricted rights and destabilize the free-choice distribution.
- In region 2, there is a paradox: starting from free-choice, players will prefer to maintain free-choice even though $a > 1/3$.
- In region 3, there is a paradox: starting from restricted rights, players will prefer to maintain

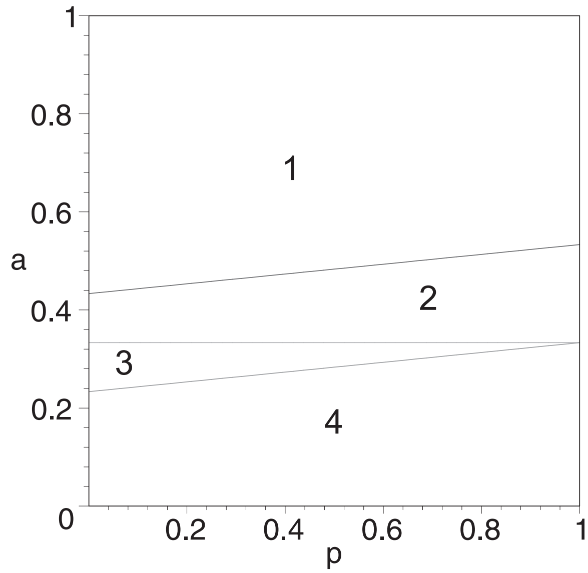

Figure 2:

restricted rights even though $a < 1/3$.

- In region 4, there is no paradox: for a sufficiently low, players will agree to destabilize restricted rights and preserve the free-choice distribution.

The inefficient restriction or preservation of rights highlighted in Remark 7 depends on the riskiness of destabilization. The larger is the gap $G - L$, the larger is the range of (a, p) values generating the paradox. Since losses of rights are weighted more heavily than gains of the same rights, players may prefer a constitutional guarantee of free-choice (in which case rights are never lost) versus a situation where a preferred right is likely to be received but possibly taken away (region 2). Similarly, they may prefer restrictions on rights rather than confront the possibility that they are assigned a preferred right, which is then lost and replaced with a non-preferred right (region 3).[8]

[8] For example, for Lewd, if $p = 0$, there is a 50% chance of being assigned $\{R\}$. When rights are redrawn, there is a 50% chance of being assigned $\{D\}$. Lewd places the weight $-L$ on this possibility. In the reverse case, Lewd places weight G on starting with $\{D\}$ and ending up with $\{R\}$. Since $G < L$, the former possibility is more vivid. For a not much lower than a_e, Lewd may prefer a stable restricted rights set of $\{D\}$ to a riskier situation.

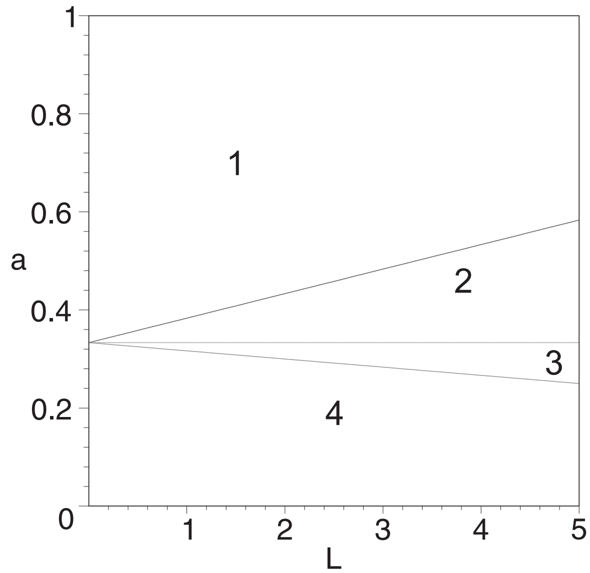

Figure 3:

Another visual characterization of the paradox is given in Figure 3. Here, p is fixed at 1/2 while a and L are permitted to vary. Incidences of the paradox are illustrated by the triangle which emanates from $(a = 1/3, L = 0)$, with $G = 0$. As the degree of loss aversion (L) grows, so does the range of a which sustains paradoxical outcomes.

4 Conclusion

The purpose of this paper has been twofold. First, to reformulate Sen's paradox using the social preferences framework of Segal and Sobel (2007), and by so doing provide an intuitive game theoretic interpretation that depends on individuals' 'nosiness'. Second, to investigate how the existence of loss aversion over rights affects pragmatic resolutions of the paradox. In our model, loss aversion can influence final outcomes by stabilizing ex ante distributions of rights (e.g. constitutional guarantees or restrictions) which generate inefficient outcomes ex post. Thus, individuals may face a behavioral constraint in overcoming or 'resolving' Sen's paradox, a result which is absent without loss aversion.

The degree to which this is so depends in turn on the degree of individuals' social preferences, or nosiness. If individuals have no social preferences, even high degrees of loss aversion will not induce inefficient distributions of rights to become enshrined constitutionally. Similarly, extremely nosy individuals may still choose to (efficiently) guarantee restrictions on rights in the face of loss aversion. The analysis demonstrates, however, that failure to overcome the paradox may also be a function of pre-existing rights distributions. This is a function of valence-framing explained by loss aversion.

Finally, the environment we have analyzed here is also amenable to experimental investigation given the reliance on the theory of interdependent preferences. Indeed, we are pursuing the empirical avenue whilst keeping our agenda constant: given behavioral game theoretic translations of social choice theory, is it possible to overcome impossibilities using psychological notions of decision-making?

References

[1] Aldrich, J. (1977), "The Dilemma of a Paretian Liberal: Some Consequences of Sen's Theorem," Public Choice, v. 30.

[2] Gaertner, Pattenaik and Suzumura (1992), "Individual Rights Revisited,"Economica, v. 59.

[3] Gibbard, A. (1974), "A Pareto-Consistent Libertarian Claim,"Journal of Economic Theory, v. 7.

[4] Levin, I. P., Schneider, S. L. and Gaeth, G. J. (1998), "All Frames Are Not Created Equal: A Typology and Critical Analysis of Framing Effects,"Organizational Behavior and Human Decision Processes, v. 76.

[5] Pattenaik, P. (1994), "On Modelling Individual Rights: Some Conceptual Issues", Social Choice Re-Examined (K. Arrow, A. Sen and K. Suzumura (eds.)), v.2, St. Martin's Press (New York).

[6] Seidl, C. (1996), "Foundations and Implications of Rights", Social Choice Re-Examined (K. Arrow, A. Sen and K. Suzumura (eds.)), v. 2, St. Martin's Press (New York).

[7] Segal, U., and Sobel, J. (2007), "Tit for tat: Foundations of Preferences for Reciprocity in Strategic Settings," Journal of Economic Theory, v. 136.

[8] Sen, A. (1970), "The Impossibility of a Paretian Liberal,"Journal of Political Economy, v. 78.

[9] Sen, A. (1976), "Liberty, Unanimity and Rights," Economica, v. 43.