Solution to Exercise 4.15

⋆4.15 P values for two-tailed tests based on statistics that have asymmetric distributions are not calculated as in Section 4.2. Let the CDF of the statistic τ be denoted as F, where $F(-x) \neq 1 - F(x)$ for general x. Suppose that, for any level α, the critical values c_α^- and c_α^+ are defined, analogously to (4.05), by the equations

$$F(c_\alpha^-) = \alpha/2 \quad \text{and} \quad F(c_\alpha^+) = 1 - \alpha/2.$$

Show that the marginal significance level associated with a realized statistic $\hat{\tau}$ is $2\min\big(F(\hat{\tau}), 1 - F(\hat{\tau})\big)$.

The realized statistic $\hat{\tau}$ is at the margin of rejection for a given level if it is equal to one or other of the critical values for that level. Thus the marginal significance level, which is the P value corresponding to $\hat{\tau}$, must satisfy one of the two equations

$$F(\hat{\tau}) = \alpha/2 \quad \text{or} \quad F(\hat{\tau}) = 1 - \alpha/2,$$

for which the solutions are $\alpha = 2F(\hat{\tau})$ and $\alpha = 2\big(1 - F(\hat{\tau})\big)$. If $F(\hat{\tau})$ is greater than one half, then $2F(\hat{\tau}) > 1$, and $2\big(1 - F(\hat{\tau})\big) < 1$. On the other hand, if $F(\hat{\tau})$ is less than one half, it is the other way around. In either case, the solution we want is the lesser of the two, as we were required to show.