This regression has two groups of regressors, as required for the application of the FWL Theorem. That theorem implies that the estimates $\hat{\boldsymbol{\beta}}$ and the residuals $\hat{\boldsymbol{u}}$ can also be obtained by running the FWL regression

$$\boldsymbol{M_S y} = \boldsymbol{M_S X\beta} + \text{residuals}, \tag{2.52}$$

where, as the notation suggests, $\boldsymbol{M_S} \equiv \mathbf{I} - \boldsymbol{S}(\boldsymbol{S}^\top\boldsymbol{S})^{-1}\boldsymbol{S}^\top$.

The effect of the projection $\boldsymbol{M_S}$ on $\boldsymbol{y}$ and on the explanatory variables in the matrix $\boldsymbol{X}$ can be considered as a form of **seasonal adjustment**. By making $\boldsymbol{M_S y}$ orthogonal to all the seasonal variables, we are, in effect, purging it of its seasonal variation. Consequently, $\boldsymbol{M_S y}$ can be called a **seasonally adjusted**, or **deseasonalized**, version of $\boldsymbol{y}$, and similarly for the explanatory variables. In practice, such seasonally adjusted variables can be conveniently obtained as the residuals from regressing $\boldsymbol{y}$ and each of the columns of $\boldsymbol{X}$ on the variables in $\boldsymbol{S}$. The FWL Theorem tells us that we get the same results in terms of estimates of $\boldsymbol{\beta}$ and residuals whether we run (2.51), in which the variables are unadjusted and seasonality is explicitly accounted for, or run (2.52), in which all the variables are seasonally adjusted by regression. This was, in fact, the subject of the famous paper by Lovell (1963).

The equivalence of (2.51) and (2.52) is sometimes used to claim that, in estimating a regression model with time-series data, it does not matter whether one uses "raw" data, along with seasonal dummies, or seasonally adjusted data. Such a conclusion is completely unwarranted. Official seasonal adjustment procedures are almost never based on regression; using official seasonally adjusted data is therefore *not* equivalent to using residuals from regression on a set of seasonal variables. Moreover, if (2.51) is not a sensible model (and it would not be if, for example, the seasonal pattern were more complicated than that given by $\boldsymbol{S\delta}$), then (2.52) is not a sensible specification either. Seasonality is actually an important practical problem in applied work with time-series data. We will discuss it further in Chapter 13. For more detailed treatments, see Hylleberg (1986, 1992) and Ghysels and Osborn (2001).

The deseasonalization performed by the projection $\boldsymbol{M_S}$ makes all variables orthogonal to the constant as well as to the seasonal dummies. Thus the effect of $\boldsymbol{M_S}$ is not only to deseasonalize, but also to center, the variables on which it acts. Sometimes this is undesirable; if so, we may use the three variables $\boldsymbol{s}_i'$ given in (2.49). Since they are themselves orthogonal to the constant, no centering takes place if only these three variables are used for seasonal adjustment. An explicit constant should normally be included in any regression that uses variables seasonally adjusted in this way.

Time Trends

Another sort of constructed, or artificial, variable that is often encountered in models of time-series data is a **time trend**. The simplest sort of time trend is the **linear time trend**, represented by the vector $\boldsymbol{T}$, with typical element