where the new parameter α is the short-run multiplier, $\delta_1 = \lambda_2 - 1$, and $\delta_2 = (1 - \lambda_2)\eta_2$. Since (14.54) is just a linear regression, the parameter of interest, which is η_2, can be estimated by $\hat{\eta}_2^{\mathrm{E}} \equiv -\hat{\delta}_2/\hat{\delta}_1$, using the OLS estimates of δ_1 and δ_2.

Equation (14.54) is without doubt an unbalanced regression, and so we must expect that the OLS estimates do not have their usual distributions. It turns out that the **ECM estimator** $\hat{\eta}_2^{\mathrm{E}}$ is a super-consistent estimator of η_2. In fact, it is usually less biased than the levels estimator $\hat{\eta}_2^{\mathrm{L}}$ obtained from the simple regression of y_{t2} on y_{t1}, as readers are invited to check by simulation in Exercise 14.21.

In the general case, with g cointegrated variables, we may estimate the cointegrating vector using the linear regression

$$\Delta y_t = \boldsymbol{X}_t\boldsymbol{\gamma} + \Delta\boldsymbol{Y}_{t2}\,\boldsymbol{\alpha} + \delta y_{t-1} + \boldsymbol{Y}_{t-1,2}\,\boldsymbol{\delta}_2 + e_t, \tag{14.55}$$

where, as before, $\boldsymbol{X}_t$ is a vector of deterministic regressors, $\boldsymbol{\gamma}$ is the associated parameter vector, $\boldsymbol{Y}_t = [y_t \quad \boldsymbol{Y}_{t2}]$ is a $1 \times g$ vector, δ is a scalar, and $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}_2$ are both $(g-1)$–vectors. Regression (14.54) is evidently a special case of regression (14.55). The super-consistent ECM estimator of $\boldsymbol{\eta}_2$ is then the ratio of the OLS estimator $\hat{\boldsymbol{\delta}}_2$ to the OLS estimator $-\hat{\delta}$.

Other Approaches

When we cannot, or do not want to, specify an ECM, at least two other methods are available for estimating a cointegrating vector. One, proposed by Phillips and Hansen (1990), is called **fully modified estimation**. The idea is to modify the OLS estimate of $\boldsymbol{\eta}_2$ in equation (14.45) by subtracting an estimate of the bias. The result turns out to be asymptotically multivariate normal, and it is possible to estimate its asymptotic covariance matrix. To explain just how fully modified estimation works would require more space than we have available. Interested readers should consult the original paper or Banerjee, Dolado, Galbraith, and Hendry (1993, Chapter 7).

A second approach, which is due to Saikkonen (1991), is much simpler to describe and implement. We run the regression

$$y_t = \boldsymbol{X}_t\boldsymbol{\gamma} + \boldsymbol{Y}_{t2}\boldsymbol{\eta}_2 + \sum_{j=-p}^{p}\Delta\boldsymbol{Y}_{t+j,2}\,\boldsymbol{\delta}_j + \nu_t \tag{14.56}$$

by OLS. Observe that regression (14.56) is just regression (14.45) with the addition of p leads and p lags of the first differences of $\boldsymbol{Y}_{t2}$. As with augmented Dickey-Fuller tests, the idea is to add enough leads and lags so that the error terms appear to be serially independent. Provided that p is allowed to increase at the appropriate rate as $n \to \infty$, this regression yields estimates that are asymptotically efficient.