a static regression model with AR(1) errors; and when $\beta_1 = 1$ and $\gamma_1 = -\gamma_0$, we have a model in first differences that can be written as

$$\Delta y_t = \beta_0 + \gamma_0 \Delta x_t + u_t.$$

Before we accept any of these special cases, it makes sense to test them against (13.59). With one important exception, this can be done by means of asymptotic t or F tests, which it may be wise to bootstrap when the sample size is not large. The exception, which will be discussed in Chapter 14, is testing the restriction that $\beta_1 = 1$.

It is usually desirable to impose the condition that $|\beta_1| < 1$ in (13.59). Strictly speaking, this is not a stationarity condition, since we cannot expect y_t to be stationary without imposing further conditions on the explanatory variable x_t. However, it is easy to see that, if this condition is violated, the dependent variable y_t exhibits explosive behavior. If the condition is satisfied, there may exist a long-run equilibrium relationship between y_t and x_t, which can be used to develop a particularly interesting reparametrization of (13.59).

Suppose there exists an equilibrium value x° to which x_t would converge as $t \to \infty$ in the absence of shocks. Then, in the absence of the error terms u_t, y_t would converge to a steady-state long-run equilibrium value y° such that

$$y^\circ = \beta_0 + \beta_1 y^\circ + (\gamma_0 + \gamma_1)x^\circ.$$

Solving this equation for y° as a function of x° yields

$$\begin{aligned} y^\circ &= \frac{\beta_0}{1-\beta_1} + \frac{\gamma_0 + \gamma_1}{1-\beta_1} x^\circ \\ &= \frac{\beta_0}{1-\beta_1} + \lambda x^\circ, \end{aligned} \tag{13.60}$$

where

$$\lambda \equiv \frac{\gamma_0 + \gamma_1}{1-\beta_1}. \tag{13.61}$$

This is the long-run derivative of y° with respect to x°, and it is an elasticity if both series are in logarithms. An estimate of λ can be computed directly from the estimates of the parameters of (13.59). Note that the result (13.60) and the definition (13.61) make sense only if the condition $|\beta_1| < 1$ is satisfied.

Because it is so general, the ADL(p, q) model is a good place to start when attempting to specify a dynamic regression model. In many cases, setting $p = q = 1$ is sufficiently general, but with quarterly data it may be wise to start with $p = q = 4$. Of course, we very often want to impose restrictions on such a model. Depending on how we write the model, different restrictions may naturally suggest themselves. These can be tested in the usual way by means of asymptotic F and t tests, which may be bootstrapped to improve their finite-sample properties.