from zero, such that $\boldsymbol{A\beta} = \mathbf{0}$. Since $k = \sum_{i=1}^{g} k_i$, we may write the vector $\boldsymbol{\beta}$ as $[\boldsymbol{\beta}_1 \vdots \ldots \vdots \boldsymbol{\beta}_g]$, where $\boldsymbol{\beta}_i$ is a k_i–vector for $i = 1, \ldots, g$. Show that there exists a nonzero $\boldsymbol{\beta}$ such that $\boldsymbol{A\beta} = \mathbf{0}$ if and only if, for at least one i, there is a nonzero $\boldsymbol{\beta}_i$ such that $\boldsymbol{P_W X}_i\boldsymbol{\beta}_i = \mathbf{0}$, that is, if $\boldsymbol{P_W X}_i$ does not have full column rank.

Show that, if $\boldsymbol{P_W X}_i$ has full column rank, then there exists a unique solution of the estimating equations (12.60) for the parameters $\boldsymbol{\beta}_i$ of equation i.

12.17 Consider the linear simultaneous equations model

$$\begin{aligned} y_{t1} &= \beta_{11} + \beta_{21} z_{t2} + \beta_{31} z_{t3} + \gamma_{21} y_{t2} + u_{t1} \\ y_{t2} &= \beta_{12} + \beta_{22} z_{t2} + \beta_{42} z_{t4} + \beta_{52} z_{t5} + \gamma_{12} y_{t1} + u_{t2}. \end{aligned} \tag{12.124}$$

If this model is written in the matrix notation of (12.68), precisely what are the matrices $\boldsymbol{B}$ and $\boldsymbol{\Gamma}$ equal to?

12.18 Demonstrate that, if each equation in the linear simultaneous equations model (12.54) is just identified, in the sense that the order condition for identification is satisfied as an equality, then the number of restrictions on the elements of the matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{B}$ of the restricted reduced form (12.70) is exactly g^2. In other words, demonstrate that the restricted and unrestricted reduced forms have the same number of parameters in this case.

12.19 Show that all terms that depend on the matrix $\boldsymbol{V}$ of error terms in the finite-sample expression for $n^{-1}\boldsymbol{X}_1^\top \boldsymbol{P_W X}_1$ obtained from equation (12.76) tend to zero as $n \to \infty$.

12.20 Consider the following $p \times q$ partitioned matrix

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{I}_m & \boldsymbol{A}_{12} \\ \mathbf{O} & \boldsymbol{A}_{22} \end{bmatrix},$$

where $m < \min(p, q)$. Show that $\boldsymbol{A}$ has full column rank if and only if $\boldsymbol{A}_{22}$ has full column rank. **Hint:** In order to do so, one can show that the existence of a nonzero q–vector $\boldsymbol{x}$ such that $\boldsymbol{Ax} = \mathbf{0}$ implies the existence of a nonzero $(q - m)$–vector $\boldsymbol{x}_2$ such that $\boldsymbol{A}_{22}\boldsymbol{x}_2 = \mathbf{0}$, and vice versa.

⋆**12.21** Consider equation (12.72), the first structural equation of the linear simultaneous system (12.68), with the variables ordered as described in the discussion of the asymptotic identification of this equation. Let the matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{B}$ of the full system (12.68) be partitioned as follows:

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{\beta}_{11} & \boldsymbol{B}_{12} \\ \mathbf{0} & \boldsymbol{B}_{22} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Gamma} = \begin{bmatrix} 1 & \boldsymbol{\Gamma}_{02} \\ -\boldsymbol{\beta}_{21} & \boldsymbol{\Gamma}_{12} \\ \mathbf{0} & \boldsymbol{\Gamma}_{22} \end{bmatrix},$$

where $\boldsymbol{\beta}_{11}$ is a k_{11}–vector, $\boldsymbol{B}_{12}$ and $\boldsymbol{B}_{22}$ are, respectively, $k_{11} \times (g-1)$ and $(l - k_{11}) \times (g-1)$ matrices, $\boldsymbol{\beta}_{21}$ is a k_{21}–vector, and $\boldsymbol{\Gamma}_{02}$, $\boldsymbol{\Gamma}_{12}$, and $\boldsymbol{\Gamma}_{22}$ are, respectively, $1 \times (g-1)$, $k_{21} \times (g-1)$, and $(g - k_{21} - 1) \times (g-1)$ matrices. Check that the restrictions imposed in this partitioning correspond correctly to the structure of (12.72).