In Exercise 12.19, readers are invited to check that everything that depends on the matrix $\boldsymbol{V}$ does indeed tend to zero in the above limit. Since we assumed that $\boldsymbol{S}_{\boldsymbol{W}^\top\boldsymbol{W}}$ is positive definite, it follows that equation (12.72) is asymptotically identified if and only if the matrix

$$\begin{bmatrix} \mathbf{I}_{k_{11}} & \boldsymbol{\Pi}_{11} \\ \mathbf{O} & \boldsymbol{\Pi}_{21} \end{bmatrix} \tag{12.77}$$

is of full column rank $k_1 = k_{11} + k_{21}$. Because this matrix has l rows, this is not possible unless $l \geq k_1$, that is, unless the order condition is satisfied. However, even if the order condition is satisfied, there can perfectly well exist parameter values for which (12.77) does not have full column rank. The important conclusion of this analysis is that asymptotic identification of an equation in a linear simultaneous system depends not only on the properties of the instrumental variables $\boldsymbol{W}$, but also on the specific parameter values of the DGP.

In Exercise 12.20, readers are asked to show that the matrix (12.77) has full column rank if and only if the $(l - k_{11}) \times k_{21}$ submatrix $\boldsymbol{\Pi}_{21}$ has full column rank. While this is a simple enough condition, it is expressed in terms of the reduced form parameters, which are usually not subject to a simple interpretation. It is therefore desirable to have a characterization of the asymptotic identification condition in terms of the structural parameters. In Exercise 12.21, notation that is suitable for deriving such a characterization is proposed, and readers are asked to develop it in Exercise 12.22.

The numerical condition that the matrix (12.66) be nonsingular is satisfied by almost all data sets, even when the rank condition for asymptotic identification is not satisfied. When this happens, the failure of that condition manifests itself as the phenomenon of **weak instruments** that we discussed in Section 8.4. In such a case, we might be tempted to add additional instruments, such as lags of the instruments themselves or other predetermined variables that may be correlated with them. But doing this cannot lead to asymptotic identification, because it would simply append columns of zeros to the matrix $\boldsymbol{\Pi}$ of reduced form coefficients, and it is obvious that such an operation cannot convert a matrix of deficient rank into one of full rank.

A discussion of asymptotic identification that is more detailed than the present one, but still reasonably compact, is provided by Davidson and MacKinnon (1993, Section 18.3). Much fuller treatments may be found in Fisher (1976) and Hsiao (1983).

Three-Stage Least Squares

The efficient GMM estimator defined by the estimating equations (12.60) is not feasible unless $\boldsymbol{\Sigma}$ is known. However, we can compute a feasible GMM estimator if we can obtain a consistent estimate of $\boldsymbol{\Sigma}$, and this is easy to do. We first estimate the individual equations of the system by generalized IV,