Here $v_{tj} \equiv -\boldsymbol{W}_{tj}\boldsymbol{\beta}^j + h_{ti(j)}$, where $h_{ti}$ denotes the inclusive value (11.39) of subset $A_i$, and $\delta_{ij}$ is the Kronecker delta.

When $\theta_k = 1$, $k = 1, \ldots, m$, the nested logit probabilities reduce to the multinomial logit probabilities (11.34). Show that, if the $\Pi_{tj}$ are given by (11.34), then the vector of partial derivatives of $\Pi_{tj}$ with respect to the components of $\boldsymbol{\beta}^l$ is $\Pi_{tj}\boldsymbol{W}_{tl}(\delta_{jl} - \Pi_{tl})$.

**⋆11.22** Explain how to use the DCAR (11.42) to test the IIA assumption for the conditional logit model (11.36). This involves testing it against the nested logit model (11.40) with the $\boldsymbol{\beta}^j$ constrained to be the same. Do this for the special case in which $J = 2$, $A_1 = \{0, 1\}$, $A_2 = \{2\}$. **Hint:** Use the results proved in the preceding exercise.

**11.23** Using the fact that the infinite series expansion of the exponential function, convergent for all real $z$, is

$$\exp z = \sum_{n=0}^{\infty} \frac{z^n}{n!},$$

where by convention we define $0! = 1$, show that $\sum_{y=0}^{\infty} e^{-\lambda}\lambda^y/y! = 1$, and that therefore the Poisson distribution defined by (11.47) is well defined on the nonnegative integers. Then show that the expectation and variance of a random variable $Y$ that follows the Poisson distribution are both equal to $\lambda$.

**11.24** Let the $n^{\text{th}}$ uncentered moment of the Poisson distribution with parameter $\lambda$ be denoted by $M_n(\lambda)$. Show that these moments can be generated by the recurrence $M_{n+1}(\lambda) = \lambda\big(M_n(\lambda) + M_n'(\lambda)\big)$, where $M_n'(\lambda)$ is the derivative of $M_n(\lambda)$. Using this result, show that the third and fourth *central* moments of the Poisson distribution are $\lambda$ and $\lambda + 3\lambda^2$, respectively.

**11.25** Explain precisely how you would use the artificial regression (11.55) to test the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$ in the Poisson regression model for which $\lambda_t(\boldsymbol{\beta}) = \exp(\boldsymbol{X}_{t1}\boldsymbol{\beta}_1 + \boldsymbol{X}_{t2}\boldsymbol{\beta}_2)$. Here $\boldsymbol{\beta}_1$ is a $k_1$–vector and $\boldsymbol{\beta}_2$ is a $k_2$–vector, with $k = k_1 + k_2$. Consider two cases, one in which the model is estimated subject to the restriction and one in which it is estimated unrestrictedly.

**⋆11.26** Suppose that $y_t$ is a count variable, with conditional mean $\mathrm{E}(y_t) = \exp(\boldsymbol{X}_t\boldsymbol{\beta})$ and conditional variance $\mathrm{E}\big(y_t - \exp(\boldsymbol{X}_t\boldsymbol{\beta})\big)^2 = \gamma^2\exp(\boldsymbol{X}_t\boldsymbol{\beta})$. Show that ML estimates of $\boldsymbol{\beta}$ under the incorrect assumption that $y_t$ is generated by a Poisson regression model with mean $\exp(\boldsymbol{X}_t\boldsymbol{\beta})$ are asymptotically efficient in this case. Also show that the OLS covariance matrix from the artificial regression (11.55) is asymptotically valid.

**11.27** Suppose that $y_t$ is a count variable with conditional mean $\mathrm{E}(y_t) = \exp(\boldsymbol{X}_t\boldsymbol{\beta})$ and unknown conditional variance. Show that, if the artificial regression (11.55) is evaluated at the ML estimates for a Poisson regression model which specifies the conditional mean correctly, the HCCME $\mathrm{HC}_0$ for that artificial regression is numerically equal to expression (11.65), which is an asymptotically valid covariance matrix estimator in this case.

**11.28** The file **count.data**, which is taken from Gurmu (1997), contains data for 485 household heads who may or may not have visited a doctor during a certain period of time. The variables in the file are: