is more than one way to formulate moment conditions like (8.11) using the available instruments. If $l = k$, the model (8.10) is said to be **just identified** or **exactly identified**, because there is only one way to formulate the moment conditions. If $l < k$, it is said to be **underidentified**, because there are fewer moment conditions than parameters to be estimated, and equations (8.11) therefore have no unique solution.

If any instruments at all are available, it is normally possible to generate an arbitrarily large collection of them, because *any* deterministic nonlinear function of the $l$ components of the $t^{\text{th}}$ row $\boldsymbol{W}_t$ of $\boldsymbol{W}$ can be used as the $t^{\text{th}}$ component of a new instrument.[1] If (8.10) is underidentified, some such procedure is necessary if we wish to obtain consistent estimates of all the elements of $\boldsymbol{\beta}$. Alternatively, we would have to impose at least $k-l$ restrictions on $\boldsymbol{\beta}$ so as to reduce the number of independent parameters that must be estimated to no more than the number of instruments.

For models that are just identified or overidentified, it is often desirable to limit the set of potential instruments to deterministic *linear* functions of the instruments in $\boldsymbol{W}$, rather than allowing arbitrary deterministic functions. We will see shortly that this is not only reasonable but optimal for linear simultaneous equation models. This means that the IV estimator is unique for a just identified model, because there is only one $k$–dimensional linear space $\mathcal{S}(\boldsymbol{W})$ that can be spanned by the $k = l$ instruments, and, as we saw earlier, the IV estimator for a given model depends only on the space spanned by the instruments.

We can always treat an overidentified model as if it were just identified by choosing exactly $k$ linear combinations of the $l$ columns of $\boldsymbol{W}$. The challenge is to choose these linear combinations optimally. Formally, we seek an $l \times k$ matrix $\boldsymbol{J}$ such that the $n \times k$ matrix $\boldsymbol{WJ}$ is a valid instrument matrix and such that, by using $\boldsymbol{J}$, the asymptotic covariance matrix of the estimator is minimized in the class of IV estimators which use an $n \times k$ matrix of instruments that are linear combinations of the columns of $\boldsymbol{W}$.

There are three requirements that the matrix $\boldsymbol{J}$ must satisfy. The first of these is that it should have full column rank of $k$. Otherwise, the space spanned by the columns of $\boldsymbol{WJ}$ would have rank less than $k$, and the model would be underidentified. The second requirement is that $\boldsymbol{J}$ should be at least asymptotically deterministic. If not, it is possible that condition (8.16) applied to $\boldsymbol{WJ}$ could fail to hold. The last requirement is that $\boldsymbol{J}$ be chosen to minimize the asymptotic covariance matrix of the resulting IV estimator, and we now explain how this may be achieved.

Since the explanatory variables $\boldsymbol{X}$ satisfy (8.18), it follows from (8.17) and (8.20) that the asymptotic covariance matrix of the IV estimator computed

[1] This procedure would not work if, for example, all of the original instruments were binary variables.