parameter, and $\boldsymbol{\gamma}$ is an r–vector of parameters. Under the null hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$, the function $h(\delta + \boldsymbol{Z}_t\boldsymbol{\gamma})$ collapses to $h(\delta)$, a constant. One plausible specification of the skedastic function is

$$h(\delta + \boldsymbol{Z}_t\boldsymbol{\gamma}) = \exp(\delta + \boldsymbol{Z}_t\boldsymbol{\gamma}) = \exp(\delta)\exp(\boldsymbol{Z}_t\boldsymbol{\gamma}).$$

Under this specification, the variance of u_t reduces to the constant $\sigma^2 \equiv \exp(\delta)$ when $\boldsymbol{\gamma} = \mathbf{0}$. Since, as we will see, one of the advantages of tests based on artificial regressions is that they do not depend on the functional form of $h(\cdot)$, there is no need for us to consider specifications less general than (7.24).

If we define v_t as the difference between u_t^2 and its conditional expectation, we can rewrite equation (7.24) as

$$u_t^2 = h(\delta + \boldsymbol{Z}_t\boldsymbol{\gamma}) + v_t, \tag{7.25}$$

which has the form of a regression model. While we would not expect the error term v_t to be as well behaved as the error terms in most regression models, since the distribution of u_t^2 is almost always skewed to the right, it does have mean zero by definition, and we will assume that it has a finite, and constant, variance. This assumption would probably be excessively strong if $\boldsymbol{\gamma}$ were nonzero, but it seems perfectly reasonable to assume that the variance of v_t is constant under the null hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$.

Suppose, to begin with, that we actually observe the u_t. Since (7.25) has the form of a regression model, we can then test the null hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$ by using a Gauss-Newton regression. Suppose the sample mean of the u_t^2 is $\tilde{\sigma}^2$. Then the obvious estimate of δ under the null hypothesis is just $\tilde{\delta} \equiv h^{-1}(\tilde{\sigma}^2)$. The GNR corresponding to (7.25) is

$$u_t^2 - h(\delta + \boldsymbol{Z}_t\boldsymbol{\gamma}) = h'(\delta + \boldsymbol{Z}_t\boldsymbol{\gamma})b_\delta + h'(\delta + \boldsymbol{Z}_t\boldsymbol{\gamma})\boldsymbol{Z}_t\boldsymbol{b}_{\boldsymbol{\gamma}} + \text{residual},$$

where $h'(\cdot)$ denotes the first derivative of $h(\cdot)$, b_δ is the coefficient that corresponds to δ, and $\boldsymbol{b}_{\boldsymbol{\gamma}}$ is the r–vector of coefficients that corresponds to $\boldsymbol{\gamma}$. When it is evaluated at $\delta = \tilde{\delta}$ and $\boldsymbol{\gamma} = \mathbf{0}$, this GNR simplifies to

$$u_t^2 - \tilde{\sigma}^2 = h'(\tilde{\delta})b_\delta + h'(\tilde{\delta})\boldsymbol{Z}_t\boldsymbol{b}_{\boldsymbol{\gamma}} + \text{residual}. \tag{7.26}$$

Since $h'(\tilde{\delta})$ is just a constant, its presence has no effect on the explanatory power of the regression. Moreover, since regression (7.26) includes a constant term, both the SSR and the centered R^2 are unchanged if we do not bother to subtract $\tilde{\sigma}^2$ from the left-hand side. Thus, for the purpose of testing the null hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$, regression (7.26) is equivalent to the regression

$$u_t^2 = b_\delta + \boldsymbol{Z}_t\boldsymbol{b}_{\boldsymbol{\gamma}} + \text{residual}, \tag{7.27}$$

with a suitable redefinition of the artificial parameters b_δ and $\boldsymbol{b}_{\boldsymbol{\gamma}}$. Observe that regression (7.27) does not depend on the functional form of $h(\cdot)$. Standard results for tests based on the GNR imply that the ordinary F statistic