the model in question simply cannot be estimated reliably with the data set we are using. Of course, some programs run the GNR (6.54) and perform the requisite checks automatically. Once we have verified that they do so, we need not bother doing it ourselves.

Computing Covariance Matrices

The second reason to run the GNR (6.54) is to calculate an estimate of $\mathrm{Var}(\hat{\boldsymbol{\beta}})$. The usual OLS covariance matrix from this regression is, by (3.50),

$$\widehat{\mathrm{Var}}(\hat{\boldsymbol{b}}) = s^2(\hat{\boldsymbol{X}}^\top\hat{\boldsymbol{X}})^{-1}, \tag{6.56}$$

where, since the regressors have no explanatory power, s^2 is the same as the one defined in equation (6.33). It is equal to the SSR from the original nonlinear regression, divided by $n-k$. Evidently, the right-hand side of equation (6.56) is identical to the right-hand side of equation (6.32), which is the standard estimator of $\mathrm{Var}(\hat{\boldsymbol{\beta}})$. Thus running the GNR (6.54) provides an easy way to calculate $\widehat{\mathrm{Var}}(\hat{\boldsymbol{\beta}})$.

Good programs for NLS estimation normally use the right-hand side of equation (6.56) to estimate the covariance matrix of $\hat{\boldsymbol{\beta}}$. Not all programs can be relied upon to do this, however, and running the GNR (6.54) is a simple way to check whether they do so and get better estimates if they do not. Sometimes, $\hat{\boldsymbol{\beta}}$ may be obtained by a method other than fully nonlinear estimation. For example, the regression function may be linear conditional on one parameter, and NLS estimates may be obtained by searching over that parameter and performing OLS estimation conditional on it. In such a case, it will be necessary to calculate the matrix (6.56) explicitly, and running the GNR (6.54) is an easy way to do so.

The GNR (6.54) can also be used to compute a heteroskedasticity-consistent covariance matrix estimate. Any HCCME for the parameters $\hat{\boldsymbol{b}}$ of the GNR is also perfectly valid for $\hat{\boldsymbol{\beta}}$. To see this, we start from the result (6.38). If $\mathrm{E}(\boldsymbol{u}\boldsymbol{u}^\top) = \boldsymbol{\Omega}$, this result implies that

$$\mathrm{Var}\big(\operatorname*{plim}_{n\to\infty} n^{1/2}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)\big) = (\boldsymbol{S}_{\boldsymbol{X}^\top\boldsymbol{X}})^{-1}\Big(\operatorname*{plim}_{n\to\infty}\frac{1}{n}\boldsymbol{X}^\top\boldsymbol{\Omega}\boldsymbol{X}\Big)(\boldsymbol{S}_{\boldsymbol{X}^\top\boldsymbol{X}})^{-1}.$$

Therefore, from the results of Section 5.5, a reasonable way to estimate $\mathrm{Var}(\hat{\boldsymbol{\beta}})$ is to use the sandwich covariance matrix

$$\widehat{\mathrm{Var}}_\mathrm{h}(\hat{\boldsymbol{\beta}}) \equiv (\hat{\boldsymbol{X}}^\top\hat{\boldsymbol{X}})^{-1}\hat{\boldsymbol{X}}^\top\hat{\boldsymbol{\Omega}}\hat{\boldsymbol{X}}(\hat{\boldsymbol{X}}^\top\hat{\boldsymbol{X}})^{-1}, \tag{6.57}$$

where $\hat{\boldsymbol{\Omega}}$ is an $n\times n$ diagonal matrix with the squared residual $\hat{u}_t^2$ as the t^{th} diagonal element. This is precisely the HCCME (5.39) for the GNR (6.54). Of course, as in Section 5.5, $\hat{\boldsymbol{\Omega}}$ can, and probably should, be replaced by a modified version with better finite-sample properties.