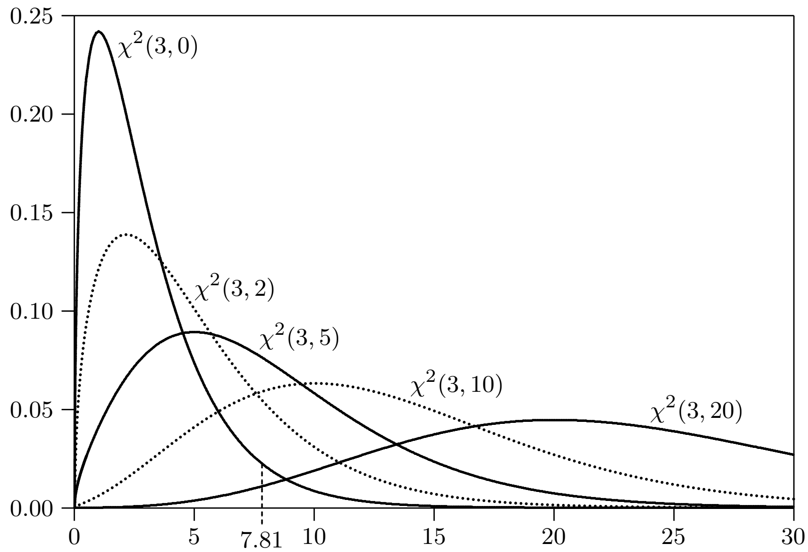

Figure 4.9 Densities of noncentral χ^2 distributions

Under the null, $\Lambda = 0$. Under either hypothesis, the distribution of the denominator of the F statistic, divided by σ^2, is central chi-squared with $n-k$ degrees of freedom, and it is independent of the numerator. The F statistic therefore has a distribution that we can write as

$$\frac{\chi^2(r, \Lambda)/r}{\chi^2(n-k)/(n-k)},$$

with numerator and denominator mutually independent. This distribution is called the **noncentral *F* distribution**, with r and $n-k$ degrees of freedom and noncentrality parameter Λ. In any given testing situation, r and $n-k$ are given, and so the difference between the distributions of the F statistic under the null and under the alternative depends only on the NCP Λ.

To illustrate this, we limit our attention to the expression (4.71), which is distributed as $\chi^2(r, \Lambda)$. As Λ increases, the distribution moves to the right and becomes more spread out. This is illustrated in Figure 4.9, which shows the density of the noncentral χ^2 distribution with 3 degrees of freedom for noncentrality parameters of 0, 2, 5, 10, and 20. The .05 critical value for the central $\chi^2(3)$ distribution, which is 7.81, is also shown. If a test statistic has the noncentral $\chi^2(3, \Lambda)$ distribution, the probability that the null hypothesis is rejected at the .05 level is the probability mass to the right of 7.81. It is evident from the figure that this probability is small for small values of the NCP and large for large ones.

In Figure 4.9, the number of degrees of freedom r is held constant as Λ is increased. If, instead, we held Λ constant, the density functions would move