E(\eta(x)) \neq \eta(E(x)). Thus, it is often very easy to calculate plims in circumstances where it would be difficult or impossible to calculate expectations.

However, working with plims can be a little bit tricky. The problem is that many of the stochastic quantities we encounter in econometrics do not have probability limits unless we divide them by \( n \) or, perhaps, by some power of \( n \).

For example, consider the matrix \( X'X \), which appears in the formula (3.04) for \( \hat{\beta} \). Each element of this matrix is a scalar product of two of the columns of \( X \), that is, two \( n \)-vectors. Thus it is a sum of \( n \) numbers. As \( n \to \infty \), we would expect that, in most circumstances, such a sum would tend to infinity as well. Therefore, the matrix \( X'X \) does not generally have a plim. However, it is not at all unreasonable to assume that

\[
\lim_{n \to \infty} \frac{1}{n} X'X = S_{X'X},
\]

where \( S_{X'X} \) is a finite nonstochastic matrix with full rank \( k \), because each element of the matrix on the left-hand side of equation (3.17) is now an average of \( n \) numbers:

\[
\left( \frac{1}{n} X'X \right)_{ij} = \frac{1}{n} \sum_{t=1}^{n} x_{ti}x_{tj}.
\]

In effect, when we write (3.17), we are implicitly making some assumption sufficient for a LLN to hold for the sequences generated by the squares of the regressors and their cross-products. Thus there should not be too much dependence between \( x_{ti}x_{tj} \) and \( x_{si}x_{sj} \) for \( s \neq t \), and the variances of these quantities should not differ too much as \( t \) and \( s \) vary.

**The OLS Estimator Is Consistent**

We can now show that, under plausible assumptions, the least-squares estimator \( \hat{\beta} \) is consistent. When the DGP is a special case of the regression model (3.03) that is being estimated, we saw in (3.05) that

\[
\hat{\beta} = \beta_0 + (X'X)^{-1}X'u.
\]

To demonstrate that \( \hat{\beta} \) is consistent, we need to show that the second term on the right-hand side here has a plim of zero. This term is the product of two matrix expressions, \((X'X)^{-1}\) and \(X'u\). Neither \( X'X \) nor \( X'u \) has a probability limit. However, we can divide both of these expressions by \( n \) without changing the value of this term, since \( n \cdot n^{-1} = 1 \). By doing so, we convert them into quantities that, under reasonable assumptions, have nonstochastic plims. Thus the plim of the second term in (3.18) becomes

\[
\left( \lim_{n \to \infty} \frac{1}{n} X'X \right)^{-1} \lim_{n \to \infty} \frac{1}{n} X'u = (S_{X'X})^{-1} \lim_{n \to \infty} \frac{1}{n} X'u = 0.
\]