

willing to transform the dependent variable as well as the independent ones, the linearity assumption can be made even less restrictive. As an example, consider the nonlinear regression model

$$y_t = e^{\beta_1} x_{t2}^{\beta_2} x_{t3}^{\beta_3} + u_t, \quad (1.23)$$

in which there are two explanatory variables,  $x_{t2}$  and  $x_{t3}$ , and the regression function is multiplicative. If the notation seems odd, suppose that there is implicitly a third explanatory variable,  $x_{t1}$ , which is constant and always equal to  $e$ . Notice that the regression function in (1.23) can be evaluated only when  $x_{t2}$  and  $x_{t3}$  are positive for all  $t$ .<sup>4</sup> It is a genuinely nonlinear regression function, because it is clearly linear neither in parameters nor in variables. For reasons that will shortly become apparent, a nonlinear model like (1.23) is very rarely estimated in practice.

A model like (1.23) is not as outlandish as may appear at first glance. It could arise, for instance, if we wanted to estimate a Cobb-Douglas production function. In that case,  $y_t$  would be output for observation  $t$ , and  $x_{t2}$  and  $x_{t3}$  would be inputs, say labor and capital. Since  $e^{\beta_1}$  is just a positive constant, it plays the role of the scale factor that is present in every Cobb-Douglas production function.

As (1.23) is written, everything enters multiplicatively except the error term. But it is easy to modify (1.23) so that the error term also enters multiplicatively. One way to do this is to write

$$y_t = e^{\beta_1} x_{t2}^{\beta_2} x_{t3}^{\beta_3} + u_t \equiv (e^{\beta_1} x_{t2}^{\beta_2} x_{t3}^{\beta_3})(1 + v_t), \quad (1.24)$$

where the error factor  $1 + v_t$  multiplies the regression function. If we now assume that the underlying errors  $v_t$  are IID, it follows that the additive errors  $u_t$  are proportional to the regression function. This may well be a more plausible specification than that in which the  $u_t$  are supposed to be IID, as was implicitly assumed in (1.23). To see this, notice first that the additive error  $u_t$  has the same units of measurement as  $y_t$ . If (1.23) is interpreted as a production function, then  $u_t$  is measured in units of output. However, the multiplicative error  $v_t$  is dimensionless. In other words, it is a pure number, like 0.02, which could be expressed as 2 percent. If the  $u_t$  are assumed to be IID, then we are assuming that the error in output is of the same order of magnitude regardless of the scale of production. If, on the other hand, the  $v_t$  are assumed to be IID, then the error is proportional to total output. This second assumption is almost always more reasonable than the first.

If the model (1.24) is a good one, the  $v_t$  should be quite small, usually less than about 0.05. For small values of the argument  $w$ , a standard approximation to

<sup>4</sup> If  $x$  and  $a$  are real numbers,  $x^a$  is not usually a real number unless  $x > 0$ . Think of the square root of  $-1$ .