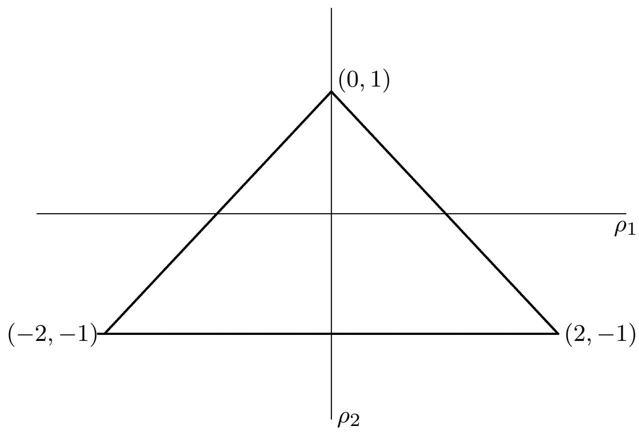

Figure 13.1 The stationarity triangle for an AR(2) process

The result (13.08) makes it clear that ρ_1 and ρ_2 are not the autocorrelations of an AR(2) process. Recall that, for an AR(1) process, the same ρ that appears in the defining equation $u_t = \rho u_{t-1} + \varepsilon_t$ is also the correlation of u_t and u_{t-1}. This simple result does not generalize to higher-order processes. Similarly, the autocovariances and autocorrelations of u_t and u_{t-i} for $i > 2$ have a more complicated form for AR processes of order greater than 1. They can, however, be determined readily enough by using the Yule-Walker equations. Thus, if we multiply both sides of equation (13.04) by u_{t-i} for any $i \geq 2$, and take expectations, we obtain the equation

$$v_i = \rho_1 v_{i-1} + \rho_2 v_{i-2}.$$

Since v_0, v_1, and v_2 are given by equations (13.08), this equation allows us to solve recursively for any v_i with $i > 2$.

Necessary conditions for the stationarity of the AR(2) process follow directly from equations (13.08). The 3×3 covariance matrix

$$\begin{bmatrix} v_0 & v_1 & v_2 \\ v_1 & v_0 & v_1 \\ v_2 & v_1 & v_0 \end{bmatrix} \tag{13.09}$$

of any three consecutive elements of an AR(2) process must be a positive definite matrix. Otherwise, the solution (13.08) to the first three Yule-Walker equations, based on the hypothesis of stationarity, would make no sense. The denominator D evidently must not vanish if this solution is to be finite. In Exercise 12.3, readers are asked to show that the lines along which it vanishes in the plane of ρ_1 and ρ_2 define the edges of a **stationarity triangle** such that the matrix (13.09) is positive definite only in the interior of this triangle. The stationarity triangle is shown in Figure 13.1.