estimates more closely, until such time as the sum of squared residuals from the GNR is within 10^{-8} of the one obtained by NLS estimation. Compare the number of iterations of this GNR-based procedure with the number used by the NLS algorithm of your software package.

6.17 Formulate a GNR, based on estimates under the alternative hypothesis, to test the restriction imposed on the model (6.94) by the model (6.95). Your test procedure should just require two OLS regressions.

6.18 Using 199 bootstrap samples, compute a parametric bootstrap P value for the test statistic obtained in Exercise 6.17. Assume that the error terms are normally distributed.

6.19 Test the hypothesis that $\gamma_0 + \gamma_1 = 0$ in (6.94). Do this in three different ways, two of which are valid in the presence of heteroskedasticity of unknown form.

6.20 For the nonlinear regression model defined implicitly by (6.95) and estimated using the data in the file **consumption.data**, perform three different tests of the hypothesis that all the coefficients are the same for the two subsamples 1953:1 to 1970:4 and 1971:1 to 1996:4. Firstly, use an asymptotic F test based on nonlinear estimation of both the restricted and unrestricted models. Secondly, use an asymptotic F test based on a GNR which requires nonlinear estimation only under the null. Finally, use a test that is robust to heteroskedasticity of unknown form. **Hint:** See regressions (6.91) and (6.92).

6.21 The original HRGNR proposed by Davidson and MacKinnon (1985a) is

$$\boldsymbol{\iota} = \acute{\boldsymbol{U}}\boldsymbol{M}_{\acute{\boldsymbol{X}}_1}\acute{\boldsymbol{X}}_2\boldsymbol{b}_2 + \text{residuals}, \tag{6.96}$$

where $\acute{\boldsymbol{U}}$, $\acute{\boldsymbol{X}}_1$, and $\acute{\boldsymbol{X}}_2$ are as defined in Section 6.8, $\boldsymbol{b}_2$ is a k_2–vector, and $\boldsymbol{M}_{\acute{\boldsymbol{X}}_1}$ is the matrix that projects orthogonally on to $\mathcal{S}^{\perp}(\acute{\boldsymbol{X}}_1)$. The test statistic for the null hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$ is n minus the SSR from regression (6.96).

Use regression (6.96), where all the matrices are evaluated at restricted NLS estimates, to retest the hypothesis of the previous question. Comment on the relationship between the test statistic you obtain and the heteroskedasticity-robust test statistic of the previous question that was based on regressions (6.91) and (6.92).

6.22 Suppose that $\boldsymbol{P}$ is a projection matrix with rank r. Without loss of generality, we can assume that $\boldsymbol{P}$ projects on to the span of the columns of an $n\times r$ matrix $\boldsymbol{Z}$. Suppose further that the n–vector $\boldsymbol{z}$ is distributed as IID$(\boldsymbol{0}, \mathbf{I})$. Show that the quadratic form $\boldsymbol{z}^\top\boldsymbol{P}\boldsymbol{z}$ follows the $\chi^2(r)$ distribution asymptotically as $n \to \infty$. (**Hint:** See the proof of Theorem 4.1.)