y_t Number of doctor visits (a nonnegative integer).
C_t Number of children in the household.
A_t A measure of access to health care.
H_t A measure of health status.

Using these data, obtain ML estimates of a Poisson regression model to explain the variable y_t, where

$$\lambda_t(\boldsymbol{\beta}) = \exp(\beta_1 + \beta_2 C_t + \beta_3 A_t + \beta_4 H_t).$$

In addition to the estimates of the parameters, report three different standard errors. One of these should be based on the inverse of the information matrix, which is valid only when the model is correctly specified. The other two should be computed using the artificial regression (11.55). One of them should be valid under the assumption that the conditional variance is proportional to $\lambda_t(\boldsymbol{\beta})$, and the other should be valid whenever the conditional mean is specified correctly. Can you explain the differences among the three sets of standard errors?

Test the model for overdispersion in two different ways. One test should be based on the OPG regression, and the other should be based on the testing regression (11.60). Note that this model is *not* the one actually estimated in Gurmu (1997).

11.29 Consider the latent variable model

$$y_t^\circ = \boldsymbol{X}_t\boldsymbol{\beta} + u_t, \quad u_t \sim \text{NID}(0, \sigma^2), \tag{11.92}$$

where $y_t = y_t^\circ$ whenever $y_t^\circ \leq y^{\max}$ and is not observed otherwise. Write down the loglikelihood function for a sample of n observations on y_t.

11.30 As in the previous question, suppose that y_t° is given by (11.92). Assume that $y_t = y_t^\circ$ whenever $y^{\min} \leq y_t^\circ \leq y^{\max}$ and is not observed otherwise. Write down the loglikelihood function for a sample of n observations on y_t.

11.31 Suppose that $y_t^\circ = \boldsymbol{X}_t\boldsymbol{\beta} + u_t$ with $u_t \sim \text{NID}(0, \sigma^2)$. Suppose further that $y_t = y_t^\circ$ if $y_t^\circ < y_t^c$, and $y_t = y_t^c$ otherwise, where y_t^c is the known value at which censoring occurs for observation t. Write down the loglikelihood function for this model.

⋆11.32 Let z be distributed as $\text{N}(0,1)$. Show that $\text{E}(z \,|\, z < x) = -\phi(x)/\Phi(x)$, where Φ and ϕ are, respectively, the CDF and PDF of the standard normal distribution. Then show that $\text{E}(z \,|\, z > x) = \phi(x)/\Phi(-x) = \phi(-x)/\Phi(-x)$. The second result explains why the inverse Mills ratio appears in (11.77).

11.33 Starting from expression (11.82) for the CDF of the Weibull distribution, show that the survivor function, the PDF, and the hazard function are as given in (11.83).