This function has exactly the same properties as an ordinary PDF. In particular, as in (1.04),

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1, x_2)\,dx_1 dx_2 = 1.$$

More generally, the probability that X_1 and X_2 jointly lie in any region is the integral of $f(x_1, x_2)$ over that region. A case of particular interest is

$$\begin{aligned} F(x_1, x_2) &= \Pr\big((X_1 \leq x_1) \cap (X_2 \leq x_2)\big) \\ &= \int_{-\infty}^{x_2}\int_{-\infty}^{x_1} f(y_1, y_2)\,dy_1 dy_2, \end{aligned} \tag{1.10}$$

which shows how to compute the CDF given the PDF.

The concept of joint probability distributions leads naturally to the important notion of **statistical independence**. Let (X_1, X_2) be a bivariate random variable. Then X_1 and X_2 are said to be **statistically independent**, or often just **independent**, if the joint CDF of (X_1, X_2) is the product of the CDFs of X_1 and X_2. In straightforward notation, this means that

$$F(x_1, x_2) = F(x_1, \infty)F(\infty, x_2). \tag{1.11}$$

The first factor here is the joint probability that $X_1 \leq x_1$ and $X_2 \leq \infty$. Since the second inequality imposes no constraint, this factor is just the probability that $X_1 \leq x_1$. The function $F(x_1, \infty)$, which is called the **marginal CDF** of X_1, is thus just the CDF of X_1 considered by itself. Similarly, the second factor on the right-hand side of (1.11) is the marginal CDF of X_2.

It is also possible to express statistical independence in terms of the **marginal density** of X_1 and the marginal density of X_2. The marginal density of X_1 is, as one would expect, the derivative of the marginal CDF of X_1,

$$f(x_1) \equiv F_1(x_1, \infty),$$

where $F_1(\cdot)$ denotes the partial derivative of $F(\cdot)$ with respect to its first argument. It can be shown from (1.10) that the marginal density can also be expressed in terms of the joint density, as follows:

$$f(x_1) = \int_{-\infty}^{\infty} f(x_1, x_2)\,dx_2. \tag{1.12}$$

Thus $f(x_1)$ is obtained by integrating X_2 out of the joint density. Similarly, the marginal density of X_2 is obtained by integrating X_1 out of the joint density. From (1.09), it can be shown that, if X_1 and X_2 are independent, so that (1.11) holds, then

$$f(x_1, x_2) = f(x_1)f(x_2). \tag{1.13}$$

Thus, when densities exist, statistical independence means that the joint density factorizes as the product of the marginal densities, just as the joint CDF factorizes as the product of the marginal CDFs.