will see, its distribution is not the same as that of an ordinary t statistic, even asymptotically. Another possible test statistic is n times the OLS estimate of $\beta - 1$ from (14.15). This statistic is called a **z statistic**. Precisely why the z statistic is valid will become clear in the next subsection. Since the z statistic is a little easier to analyze than the τ statistic, we focus on it for the moment.

The z statistic from the test regression (14.15) is

$$z = n\frac{\sum_{t=1}^{n} y_{t-1}\Delta y_t}{\sum_{t=1}^{n} y_{t-1}^2},$$

where, for ease of notation in summations, we suppose that y_0 is observed. Under the null hypothesis, the data are generated by a DGP of the form

$$y_t = y_{t-1} + \sigma\varepsilon_t, \tag{14.16}$$

or, equivalently, $y_t = y_0 + \sigma w_t$, where w_t is a standardized random walk defined in terms of ε_t by (14.01). For such a DGP, a little algebra shows that the z statistic becomes

$$z = n\frac{\sigma^2\sum_{t=1}^{n} w_{t-1}\varepsilon_t + \sigma y_0 w_n}{\sigma^2\sum_{t=1}^{n} w_{t-1}^2 + 2y_0\sigma\sum_{t=1}^{n} w_{t-1} + ny_0^2}. \tag{14.17}$$

Since the right-hand side of this equation depends on y_0 and σ in a nontrivial manner, the z statistic is not pivotal for the model (14.16). However, when $y_0 = 0$, z no longer depends on σ, and it becomes a function of the random walk w_t alone. In this special case, the distribution of z can be calculated, perhaps analytically and certainly by simulation, provided we know the distribution of the ε_t.

In most cases, we do not wish to assume that $y_0 = 0$. Therefore, we must look further for a suitable test statistic. Subtracting y_0 from both y_t and y_{t-1} in equation (14.14) gives

$$\Delta y_t = (1-\beta)y_0 + (\beta-1)y_{t-1} + \sigma\varepsilon_t.$$

Unlike (14.15), this regression has a constant term. This suggests that we should replace (14.15) by the test regression

$$\Delta y_t = \gamma_0 + (\beta-1)y_{t-1} + e_t. \tag{14.18}$$

Since $y_t = y_0 + \sigma w_t$, we may write $\boldsymbol{y} = y_0\boldsymbol{\iota} + \sigma\boldsymbol{w}$, where the notation should be obvious. The z statistic from (14.18) is still $n(\hat\beta - 1)$, and so, by application of the FWL theorem, it can be written under the null as

$$z = n\frac{\sum_{t=1}^{n}(\boldsymbol{M_\iota y})_{t-1}\Delta y_t}{\sum_{t=1}^{n}(\boldsymbol{M_\iota y})_{t-1}^2} = n\frac{\sum_{t=1}^{n}(\boldsymbol{M_\iota y})_{t-1}\sigma\varepsilon_t}{\sum_{t=1}^{n}(\boldsymbol{M_\iota y})_{t-1}^2}, \tag{14.19}$$